AHP Excel Template with multiple Inputs

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Overview (latest changes in red)

The AHP template works under Excel version MS Excel 2013. The workbook consists of 20 input worksheets for pair-wise comparisons, a sheet for the consolidation of all judgments, a summary sheet to display the result, a sheet with reference tables (random index, limits for geometric consistency index GCI, judgment scales) and a sheet for solving the eigenvalue problem when using the eigenvector method (EVM).

Limitations

- Maximum number of criteria: 10
- Maximum number of decision makers/participants: 20

Results

The result table will show all criteria with calculated weights and errors, using the EVM:

	Criterion	Comment	Weights	+/-
1	Crit-1		19.2%	1.8%
2	Crit-2		63.4%	6.1%
3	Crit-3		17.4%	1.7%
4			0.0%	0.0%
5			0.0%	0.0%
6			0.0%	0.0%
7			0.0%	0.0%
8			0.0%	0.0%
9		for 9&10 unprotect the input sheets and expand the	0.0%	0.0%
10		question section ("+" in row 66)	0.0%	0.0%

On top of the table you find a check field showing the convergence of the EVM calculation using the power method. "Iterations" shows the number of iterations needed. The value "EVM check" should be close to zero.

Thresh:	1E-08	Iterations:	9	EVM check:	1.9E-09

Below you see the Eigenvalue (Lambda), Mean Relative error (MRE) of the weights, Geometric consistency index (GCI), Ordinal inconsistency (Psi), and Consistency ratio (CR). (see annex)

Eigenvalue			Lambda:	3.009		MRE:	9.6%
Consistency Ratio	0.37	GCI: 0.03	Psi:	0.0%	CR:	1.0%	

In the section below the comparison matrix is displayed:

Matrix		Criterion 1 1	Criterion 2 2	Criterion 3 3	0 4	0 5	0 6	0 7	0 8	0 9	0 10	normalized principal Eigenvector
Criterion 1	1	1	5	1/3	-	-	-	-	-	-	-	27,9%
Criterion 2	2	1/5	1	1/7	-	-	-	-	-	-	-	7,2%
Criterion 3	3	3	7	1	-	-	-	-	-	-	-	64,9%
0	4	-	-	-	1	-	-	-	-	-	-	0,0%
0	5	-	-	-	-	1	-	-	-	-	-	0,0%
0	6	-	-	-	-	-	1	-	-	-	-	0,0%
0	7	-	-	-	-	-	-	1	-	-	-	0,0%
0	8	-	-	-	-	-	-	-	1	-	-	0,0%
0	9	-	-	-	-	-	-	-	-	1	-	0,0%
0	10	-	-	-	-	-	-	-	-	-	1	0,0%

How to use the template

1. Open the Excel file AHPcalc version dd.mm.yy.xls
2. Select the worksheet "Summary"
3. Input values in the green fields only:

n=	3	Number of criteria (3 to 10)	Scale:	1	Linear

a) Number of criteria in "n=" (2-10)
b) Scale: selected AHP scale (see annex) – default is scale 1, standard linear 1 to 9 AHP scale

1 = AHP 1-9 scale
2 = Logarithmic
3 = Square root
4 = Invers linear
5 = Balanced-n (corrected balanced scale)
6 = Power
7 = Geometric
8 = Adaptive

Note: a) The most often used scale is the original linear 1 to 9 AHP scale.
b) From version 12.08.13 onwards decimals as input values for pairwise comparisons are accepted.

N=	1	Number of Participants (1 to 20)	α:	0.1	Consensus:	n/a

c) Number of participants "N=" (1 - 20)
d) Alpha (α): threshold for acceptance of inconsistency. We recommend a value of 0.1.

Note: The consensus field is an output field showing the AHP consensus index (see annex), if you have more than one decision maker/participant. The consensus indicator ranges from 0% (no consensus between decisions makers) to 100% (full consensus between decision makers).

p=	1	selected Participant (0=consol.)	13	7	Participant 1

e) Selected participant p – default "1"
For more than 1 participant you can select whose participant's result to be displayed. Participants are numbered from 1 to 20 according the input sheets for pair-wise comparisons. When selecting 0, the consolidated result for all participants will be shown, using the geometric mean of all decision matrices.

Objective	Calculate weights with pairwise comparisons						
Author	Klaus						
Date	15-Sep-18	Thresh:	1E-08	Iterations:	9	EVM check:	1.9E-09

f) Objective (text) to describe the project/category
g) Author (text, optional)
h) Date (date, optional)

i) The table allows you to input the name of criteria and a comment for each criterion.

Criterion 1	First Criterion
Criterion 2	Second Criterion
Criterion 3	Third Criterion

Pairwise comparisons

1. Select worksheet “In1”
In each input sheet you can specify the name of the decision maker/participant, a weight for his evaluation and a date.

Participant 1	1	
Name	Weight	Date

A weight higher than one – for example two – means that his input is weighted twice the input of all other participants. The elements of the consolidated decision matrix (all participants) are calculated as *weighted* geometric mean of all individual participants (see annex).

The table below is the input table for pair-wise comparisons

		Criteria		more important ?	Scale
i	j	A	B	A or B	(1-9)
1	2	Criterion 1	Criterion 2	A	5
1	3		Criterion 3	B	3
1	4				
1	5				
1	6				
1	7				
1	8				
2	3	Criterion 2	Criterion 3	B	7
2	4				

For 3 criteria the first comparison is criterion 1 versus criterion 2. In the second last column the participant has to select either **A** (criterion 1 more important than 2), or **B** (criterion 2 more important than 1). A or B are not case sensitive. In the last column of the table the participant specifies the intensity – how much more important is 1 compared to 2 resp. 2 compared to 1. Valid inputs are integers from 1 to 9.

Important Note: If you use more than 8 criteria, you have to unprotect the input sheets and expand the lines from 49 to 65 to complete all comparisons. After unprotecting click on the “+”

At the bottom of the page the explanation of intensities (scale) is shown:

Intensity	Definition	Explanation
1	Equal importance	Two elements contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favor one element over another
5	Strong Importance	Experience and judgment strongly favor one element over another
7	Very strong importance	One element is favored very strongly over another, it dominance is demonstrated in practice
9	Extreme importance	The evidence favoring one element over another is of the highest possible order of affirmation
2,4,6,8 can be used to express intermediate values		

The next comparison is then criterion 1 versus 2, followed by 2 versus 3. For more the 3 criteria automatically more pairs will be listed in the table. When doing the comparisons, it might happen that 3 lines will be highlighted:

A	9	1	A4
A	8		
A	7	3	A9
A	6	2	A3
A	5		

This is an indication of inconsistent inputs. The most inconsistent judgment is marked with “1”. The text field after the marking shows the ideal, most consistent judgment (A4, A9 and A3 in the example above). Participants might slightly modify the highlighted judgments in direction of the ideal judgment, in order to improve consistency.

α: 0.1 CR: 32%
Consistency Ratio

After reviewing all answers, ideally no line will be highlighted and consistency is within the given threshold to make the result reliable. In addition to the consistency ratio, errors for each weights are indicated. It can happen that even with a consistency ratio below 10%, errors are significant, and some weights are overlapping within the error range.

Note: Each input sheets will show the resulting priorities calculated from the pairwise comparisons based on the row geometric mean method (RGMM). **The final calculation using the Eigen vector method (EVM) will only be shown in the summary sheet.**

n	Criteria	Comment	RGMM	+/-
1	Crit-1		19.2%	1.8%
2	Crit-2		63.4%	6.1%
3	Crit-3		17.4%	1.7%
4				

2. For more than 1 participant select worksheet “In2 … InN” and input name, date and the pairwise comparisons for additional participants.

Go back to sheet “Summary” to see the result.

Please make a reference to the author, when using the template in your work:

Goepel, Klaus D. (2013). Implementing the Analytic Hierarchy Process as a Standard Method for Multi-Criteria Decision Making In Corporate Enterprises – A New AHP Excel Template with Multiple Inputs, *Proceedings of the International Symposium on the Analytic Hierarchy Process 2013,* p 1 -10

For questions, feedback, suggestions please contact the author under https://bpmsg.com

Under https://bpmsg.com you will also find other AHP online tools for the calculation of priorities and the handling of complete AHP hierarchies and evaluation of alternatives.

Annex - Mathematical relations and formulas used

A. Scales

Intensities x, with x = 1 to 9 (integer) are transformed into c using following relations:

1- Linear $c = x$

2- Logarithmic $c = \log_2(x+1)$

3- Root square $c = \sqrt{x}$

4- Inverse linear $c = 9/(10-x)$

5- Balanced-n $c = \frac{9+(n-1)x}{9+n-x}$

6- Power $c = x^2$

7- Geometric $c = 2^{x-1}$

8- Adaptive $c = x^{\left(1+\frac{\ln(n-1)}{\ln 9}\right)}$

C, resp. $1/c$, is then used as element in the pair-wise comparison matrix.

Goepel, K. D. (2017). Comparison of judgment scales of the analytical hierarchy process - a new approach, submitted for consideration in *International Journal of Information Technology and Decision Making* © 2017 World Scientific Publishing Company http://www.worldscientific.com/worldscinet/ijitdm

Goepel, K.D. (2018). Judgment scales of the analytical hierarchy process – the balanced scale. *Proceedings of the International Symposium on the Analytic Hierarchy Process*, Hong Kong, July 2019

B. RGMM

Priorities p_i in each *input sheet* are calculated using the row geometric mean method (RGMM). With the pairwise NxN comparison matrix $\mathbf{A} = a_{ij}$

We calculate
$$r_i = \exp\left[\frac{1}{N}\sum_{j=1}^{N}\ln(a_{ij})\right] = \left(\prod_{i=1}^{N} a_{ij}\right)^{1/N}$$

and normalize:
$$p_i = r_i ./ \sum_{i=1}^{N} r_i$$

C. Inconsistencies

To find the most inconsistent comparison, we look for the pair i,j with

$$\max\left(\varepsilon_{ij} = a_{ij}\frac{p_j}{p_i}\right)$$

Consistency ratios are calculated in all *input sheets* and in the *summary sheet*. With $\lambda_{\max}$ the calculated principal eigenvalue - either based on the priority eigenvector derived from RGMM in the input sheet or derived from EVM in the summary sheet – the consistency index CI is given as

$$CI = \frac{(\lambda_{\max} - N)}{N-1}$$

The consistency *ratio CR* is calculated using $CR = \frac{CI}{RI}$

We use the Alonson/Lamata linear fit resulting in CR:

$$CR = \frac{\lambda_{\max} - N}{2.7699N - 4.3513 - N}$$

Alonso, Lamata, (2006). Consistency in the analytic hierarchy process: a new approach. *International Journal of Uncertainty, Fuzziness and Knowledge based systems, Vol 14, No 4, 445-459*

Geometric consistency index GCI is calculated using:

$$CGI = \frac{2\sum_{i<j} \ln a_{ij} - \ln \frac{p_i}{p_j}}{(N-1)(N-2)}$$

Dissonance (Ordinal Inconsistency Psi):

$$\psi_{ij} = \frac{1}{n-2}\sum_k step(-\log(a_{ij})\log(a_{ik}a_{kj}) > 1$$

where i ≠ k ≠ j and the step function is defined as:

$$step(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Overall dissonance:

$$\Psi = \frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\psi_{ij}$$

Sajid Siraj (2011). Preference elicitation from pairwise comparisons in multi-criteria decision making, Dissertation, The University of Manchester, Dec. 2010.

D. Error calculation

Eigenvector method (EVM)

$$\Delta w_i = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}\left(\frac{n}{\lambda_{\max}} a_{ik}w_k - w_i\right)^2}, i = 1, \dots, n$$

Mean relative error *MRE*

$$\left(\frac{\Delta w_i}{w_i}\right)_{\text{mean}} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(\frac{\Delta w_i}{w_i}\right)^2}$$

Row geometric mean method (RGMM)

$$\Delta_i = C\sqrt{\frac{1}{n-1}\sum_{k=1}^{n}\ln^2(a_{ik}\frac{w_k^*}{w_i^*})}$$

$$w_i = w_i^*\cosh(\Delta_i)$$

$$\Delta w_i = w_i^*\sinh(\Delta_i)$$

Tomashevskii, I. L. (2014). Geometric mean method for judgement matrices: formulas for errors, arXiv:1410.0823v1 [math.OC].

Tomashevskii, I. L. (2015). Eigenvector ranking method as a measuring tool: formulas for errors, *European Journal of Operational Research*, Volume 240, Issue 3, 1 February 2015, Pages 774-780.

E. Aggregation of individual judgments (Consolidation of participants)

The consolidated decision matrix **C** (selected participant "0") combines all *k* participants' inputs to get the aggregated group result. We use the *weighted geometric mean* of the decision matrices elements $a_{ij(k)}$ using the individual decision maker's weight w_k as given in the input sheets:

$$c_{ij} = \exp\frac{\sum_{k=1}^{N} w_k \ln a_{ij(k)}}{\sum_{k=1}^{N} w_k}$$

F. AHP consensus indicator

AHP consensus is calculated in the *summary sheet* based on the RGMM results of all inputs using *Shannon alpha and beta entropy*. The consensus indicator ranges from 0% (no consensus between decisions makers) to 100% (full consensus between decision makers).

*AHP consensus indicator S**

$$S^* = \left[M - \exp(H_{\alpha\min})/\exp(H_{\gamma\max})\right] / \left[1 - \exp(H_{\alpha\min})/\exp(H_{\gamma\max})\right]$$

with $M = 1/\exp(H_\beta)$.

$H_{\alpha,\beta,\gamma}$ is the α, β, γ Shannon entropy for the priorities of all K decision makers/participants.

Shannon alpha entropy $$H_\alpha = \frac{1}{K}\sum_{j=1}^{K}\sum_{i=1}^{N} - p_{ij} \ln p_{ij}$$

Shannon gamma entropy $$H_\gamma = \sum_{j=1}^{K} - \bar{p}_j \ln \bar{p}_j$$

with $$\bar{p}_j = \frac{1}{N}\sum_{i=1}^{N} p_{ij}$$

Shannon beta entropy $$H_\beta = H_\gamma - H_\alpha$$

We need to adjust for the maximum score c_{max} of the AHP scale used

and $$H_{\alpha\min} = -\frac{c_{\max}}{N + c_{\max} - 1}\ln\left(\frac{c_{\max}}{N + c_{\max} - 1}\right) - (N-1)\frac{1}{N + c_{\max} - 1}\ln\frac{1}{N + c_{\max} - 1}$$

$$H_{\gamma\max} = \ln(n)$$

N number of criteria, *K* number of decision makers/participants.

Interpretation of AHP consensus indicator S^*

S^*	Consensus
≤ 50%	Very low
50% - 65%	low
65% - 75%	moderate
75% - 85%	high
≥85%	Very high

For more information see: Goepel, Klaus D., Implementing the analytic hierarchy process as a standard method for multi-criteria decision making in corporate enterprises – a new AHP excel template with multiple inputs. *Proceedings of the international symposium on the analytic hierarchy process*, Kuala Lumpur, Malaysia, 2013 (Submitted Feb. 2013).