Group Consensus Cluster Analysis using Shannon Alpha- and Beta Entropy

Klaus D. Goepel, 2022-04-28
Submitted for publication

Abstract

Group decision making using multi-criteria decision methods (MCDM) is a common way to support rational decision making in various fields of applications. The input of individual group members have to be aggregated resulting in a consolidated outcome. The author introduces a consensus indicator as a measure of agreement among decision makers using Shannon entropy. Based on the concept of diversity in ecology, the partitioning of Shannon entropy into alpha and beta components is used to develop a simple cluster algorithm to identify possible subgroups of decision makers with higher consensus. This allows a deeper insight into the decision making and the group results. The process is demonstrated with two typical examples using group data as derived from the analytic hierarchy process (AHP). The algorithm is implemented and published in PHP and available as open source.

Keywords: group decision making, group consensus, Shannon entropy, cluster analysis.

1. Introduction

Multi-criteria decision making (MCDM) methods are widely applied to support the process of rational decision making; often these methods are also used to collect inputs of a group of decision makers (DM) and consolidate their judgments for group decision making.[1–3] Several group aggregation techniques are available.[4–7] In the following we assume to have a set of of criteria or categories c_i with weights w_{ik} corresponding to preferences of an individual DM k. In addition we use, without limitation of general validity for our further considerations, the simple technique of aggregation of individual priorities (AIP) by means of the arithmetic mean.[8,9]

Although mathematically it is always possible to calculate an aggregated group result, it does not make sense in all cases. Consider the following simple example: We have two DM with totally opposite judgments for two criteria; the aggregation will result in equal weights (50/50) for both criteria. In fact, there is no agreement between the two decision makers, and equal weights may result in a deadlock situation.

Therefore, it is necessary to analyse the group outcome and find a *measure of consensus* (agreement) for the aggregated group result.[10–12]

We interpret our set of categories c_i with weights w_{ik} as a weight distribution for the individual DM k, and we need to find its (dis)similarity with the weight distribution w_{il} over the same set of categories of another DM l. A suitable measure of differences or similarities between distributions is relative homogeneity, based on the mathematical concept of diversity as used in ecology and biology. Instead of species distributions (relative abundance of species) in different habitats, we analyse the relative weight distribution of criteria among different decision makers.

2. Methodology

Originating from information theory, the concept of Shannon entropy is well established in biology for the measurement of biodiversity.[13] We introduce Shannon entropy and its partitioning into two independent components, alpha and beta entropy, to derive a consensus indicator for group decision making.

2.1 Shannon Entropy and Diversity

Shannon entropy H (Shannon, 1949) can be written as[14]

$$H = \sum_{i=1}^{n} -w_i \ln(w_i) \quad (1)$$

with w_i relative weight of category i

$$\sum_{i}^{n} w_i = 1. \quad (2)$$

For categories with $w_i = 0$ the contribution to H is zero. Practically H can be interpreted as a measure of the evenness of priorities among the criteria for an individual DM; the higher priorities are concentrated on fewer criteria, the lower the entropy.

Using the exponential function we get ^1D, the Hill number of order one.[15]

$$^1D = \exp(H) \quad (3)$$

In Ecology ^1D is interpreted as the effective number of species;[16] in our context it can be interpreted as the effective number of criteria. For an equal distribution of priorities across all criteria, ^1D equals the number of criteria n, and the Shannon entropy H equals $\ln(n)$. For priority given to only one single criterion, diversity ^1D is unity, and Shannon entropy H equals $\ln(1) = 0$. Roughly, ^1D measures the number of 'common' (or 'typical') criteria in a group.[17]

It is important to note that Shannon entropy, as defined in eq.(1) and (2), cannot be used as a measure of consensus, because any permutation of a given weight distribution over categories will have the same entropy value associated with it. Entropy is constant regardless of the order of the categories within the distribution. This is exactly in opposition to the requirements essential to a consensus measure.[18] We need to partition Shannon entropy into its independent two components.

2.2 Partitioning Shannon Entropy in Alpha and Beta Components

Whittaker described three terms for measuring biodiversity over spatial scales: alpha, beta, and gamma diversity.[19] Alpha diversity is the diversity within a particular area or ecosystem; beta diversity is a comparison of diversity between ecosystems and gamma diversity is a measure of the overall diversity within a large region. Using the concept of diversity allows us to partition Shannon entropy into two independent components, alpha- and beta diversity.[20]

$$H_\beta = H_\gamma - H_\alpha \tag{4}$$

Shannon γ entropy is
$$H_\gamma = \sum_{i=1}^{n} -w_{i,avg} \ln\left(w_{i,avg}\right) \tag{5}$$

with
$$w_{i,avg} = \frac{1}{k}\sum_{j=1}^{k} w_{ij} \tag{6}$$

for participants $j = 1 \dots k$ and categories $i = 1 \dots n$.

Shannon alpha entropy for a group of k decision makers is the average Shannon entropy of all individual DM.

Shannon α entropy
$$H_\alpha = \frac{1}{k}\sum_{j=1}^{k}\sum_{i=1}^{n} -w_{ij}\ln\left(w_{ij}\right) \tag{7}$$

for participants $j = 1 \dots k$ and categories $i = 1 \dots n$.

We introduce alpha diversity
$$D_\alpha = \exp\left(H_\alpha\right) \tag{8}$$

and from eq. (3) and eq. (4) we write beta diversity as

$$D_\beta = D_\gamma / D_\alpha \tag{9}$$

A high beta diversity shows a low similarity of the priorities among group members.

2.3 Relative Homogeneity, Similarity and Group Consensus

The reciprocal of beta diversity eq. (9) is a simple homogeneity measure.[21]

$$M = \frac{1}{D_\beta} = \frac{D_\alpha}{D_\gamma} \tag{10}$$

It can be transformed into a relative index of homogeneity in the range from zero to unity with

$$S = \frac{1/D_\beta - 1/n}{1 - 1/n} \tag{11}$$

using the fact that $D_{\alpha,min} = 1$ and $D_{\gamma,max} = n$.

When using the analytic hierarchy process (AHP),[22] the minimum alpha entropy $H_{\alpha,\min}$ and maximum gamma entropy $H_{\gamma,\max}$ are functions of the maximum scale value m ($m = 9$ for the fundamental AHP scale) and the number of criteria n.[23]

$$H_{\alpha,min} = -\left(\frac{m}{n+m-1}\right)\ln\left(\frac{m}{n+m-1}\right) - \left(\frac{n-1}{n+m-1}\right)\ln\left(\frac{1}{n+m-1}\right) \quad (12)$$

and $$H_{\gamma,max} = \ln(n) \quad (13)$$

With $$c = \frac{\exp(H_{\alpha,min})}{n} \quad (14)$$

we get instead of eq. (11) $$S_{AHP} = \frac{1/D_\beta - c}{1-c}. \quad (15)$$

The relative index of homogeneity S (resp. S_{AHP}) is used as our consensus indicator; it is zero, when the priorities of all DM are completely distinct, and unity when the priorities of all DM are identical.[12,23]

We now can use our simple example of two decision makers with opposite judgment, given in the introduction, to calculate the consensus. We have two criteria c_1 and c_2; DM_1 gives the preference of w_{11} = 100 % to c_1 and w_{21} = 0 % to c_2. for DM_2 w_{12} = 0 % and w_{22} = 100 % (opposite judgment). The average weight $w_{i,\mathrm{avg}}$ in eq. (6) is $w_{i,\mathrm{avg}} = 0.5$, as a group result both criteria get the same weight of 50%. Gamma entropy in eq.(5) is H_γ =-ln(1/2) = 0.6932, or gamma diversity $D_\gamma = 2$. Shannon alpha entropy is according to eq.(7) the average of the individual entropies. For both DM $H_{\alpha,j}$ = -ln (1) = 0, and therefore alpha diversity D_α = exp(0) = 1. Beta entropy eq. (4) is $H_\beta = H_\gamma - H_\alpha = 0.6932$, and beta diversity $D_\beta = D_\gamma/D_\alpha = 2$. In this simple numerical example relative homogeneity S *i*n eq.(11) calculates to (0.5 – 0.5)/(1 – 0.5) = 0; there is no consensus.

2.4 The Similarity Matrix

We calculate the consensus indicator S_{ij} for all possible pair combinations of DM i and j and arrange them in a k by k matrix M.

$$\mathrm{M}_S = \begin{matrix} 1 & S_{01} & \cdots & S_{0k} \\ S_{10} & \ddots & \cdots & \cdots \\ \cdots & \cdots & \ddots & S_{2k} \\ S_{k0} & \cdots & \cdots & 1 \end{matrix} \quad (17)$$

The diagonal of the matrix is always one, as the consensus between same decision makers is 100% and $S_{ij} = S_{ji}$. In order to identify clusters of higher consensus between subgroups of DM we will rearrange the matrix columns and rows in a way that clusters of high consensus form rectangles along the diagonal using the cluster algorithm.

2.5 Cluster algorithm

In a first step of the algorithm we define an arbitrary threshold *th* (*e.g. th* = 95%) and count the number of elements in each row of the similarity matrix M exceeding the given threshold. The row with the highest count determines the first cluster. If the count is zero for all rows, we lower the threshold by a given step size (*e.g.* 2.5%) and repeat the counting.

Once we have found the first cluster, we delete all elements of the matrix (*i.e.* set them to zero) containing DM of the first cluster CL_1. In addition we calculate the group consensus S_{CL1} of the whole cluster using eq. (5, 7) and (11) for the DM in the cluster. We then repeat the counting for the next possible cluster CL_2, until none of the remaining rows contains elements exceeding the given threshold. Decision makers belonging to these remaining matrix elements are put into a list of 'unclustered' (dissociated) DMs.

We continue with the next threshold value to scan a whole range of thresholds between th_{max} and th_{min}. This way we generate a threshold table, showing the number of clusters m_{cl} and number of dissociated participants m_{uc} as a function of the threshold th.

In a second step the algorithm will then select, based on the generated threshold table, the optimal clustering. The most simple optimization function is to find the minimum of $m_{cl} + m_{uc}$ with the condition $m_{cl} > 1$ (more than one clusters) and $m_{uc} < 3$ (one or two dissociated DM). In addition, the consensus of the first cluster found, S_{CL1} is compared with the consensus of the whole group S and a minimum defined threshold S_{min}. The reason is that the clusters should have a higher consensus than the whole group, and there should be a minimum consensus within the cluster. Therefore, only when $S_{CL1} > S$ and $S_{CL1} > S_{min}$, the threshold is proposed for clustering. Depending on the actual group data and boundary conditions there is the possibility that no clusters are found, *e.g.* when $m_{cl} = 0$ for the whole threshold range, or no improvement of $S_{CL1} > S$ can be achieved.

When a solution is found, rows and columns of the similarity matrix M are rearranged according to the clusters found, elements are colour coded, and the matrix is displayed to the user.

3. Implementation

The cluster algorithm was implemented in PHP (7.3.33), a server side scripting language.[24] The algorithm is coded as an object class and has four essential functions:

- `cluster(th)` is the main cluster algorithm: it searches for all clusters with the given threshold value *th* and returns a list of clusters and a list of dissociated DM.
- `calcThreshold()` calculates the threshold table; it returns a table with the number of clusters, the number of dissociated DM and the consensus for each threshold in the range from th_{min} to th_{max}.
- `findThreshold()` searches for the optimal threshold value to be used for final clustering.
- `calcGroupSim(cluster)` calculates alpha, beta and gamma entropy, as well as the relative homogeneity S or the consensus indicator S_{AHP}.

Project data can be imported as Java script object notation (JSON) or coma separated value (CSV) text files. Once data are uploaded, the algorithm will run and output the data input parameter, the threshold table, the consensus threshold determined for clustering, and for each cluster the members (DM) and a diagram of the averaged weights over categories. Finally, for visualization, the similarity matrix is displayed.

For the calculation of the threshold table, the threshold is varied from 97.5 % to 70 % in steps of 2.5 %. S_{min} is set to the middle of the moderate range (68.75 %). In addition to the conditions described under 2.5, we also allow one or two clusters and one or two dissociated DM capture 'outliners'.

3.1 Consensus word scale

To make the results easier to interpret, we define a descriptive word scale for the consensus range from zero to unity. For this we analyzed the consensus within 140 hierarchy nodes (a set of criteria or sub-criteria within a decision hierarchy) of 35 AHP group decision projects. It could be shown that the consensus S_{AHP} is normal distributed with a mean value of 64 % ± 3 %. With a 99.5% probability the consensus of all projects lies between 28 % and 99 %. Therefore we divided the range of the scale in four equal segments from 50 % to 100 % (going from 'low' to 'very high'), and defined the consensus for values below 50 % as 'very low'.

Table 1: Qualitative wording scale for AHP consensus indicator

Consensus S_{AHP}	0% ... 50%	50% ... 62.5%	62.5% ... 75%	75% ... 87.5%	87.5% ... 100%
Wording scale	Very low	low	moderate	high	Very high

Switching from the consensus indicator S_{AHP} to the relative homogeneity S shifts the mean value from 64 % to 70 %, which can be explained by the fact that in AHP we have a limited 1 to 9 scale and $H_{\alpha,min}$ is a function of the maximum scale value (eq(12)). As we only make a relative comparison of $S_{CL} > S_{min}$ and $S_{CL} > S$, the actual limits between the percentages for the word scale have no impact on the results.

4. Results and Discussion

After implementation the algorithm was tested with 21 AHP projects on 57 hierarchy nodes, where a group of decision makers (participants) had evaluated criteria using pairwise comparisons. We are not going into the specific details of the projects, as we just want to demonstrate the clustering, and how it can give a better insight into the aggregated group results. The group size varied from seven to 122 participants with the number of criteria between two and nine for hierarchy nodes (using S_{AHP}) and 21 to 65 for global priorities (using S).

Out of the 57 data samples the majority could be clustered in two or three clusters. For five data sets more than three clusters were necessary and for eight samples clustering was not possible. The consensus of the whole group before clustering varies from 37.2 % to 82.5 %. Figure 1 shows the average consensus of clusters after clustering as a function of the consensus of the whole group. Each data point is a averaged sum of projects in a group consensus intervall of 5 %.

Especially for projects with a low or very low consensus there is a siginificant improvement of the consensus within clusters, which indicates that the group result does not reflect the distinct judgments of group members within the subgroups. In the following we give two typical examples.

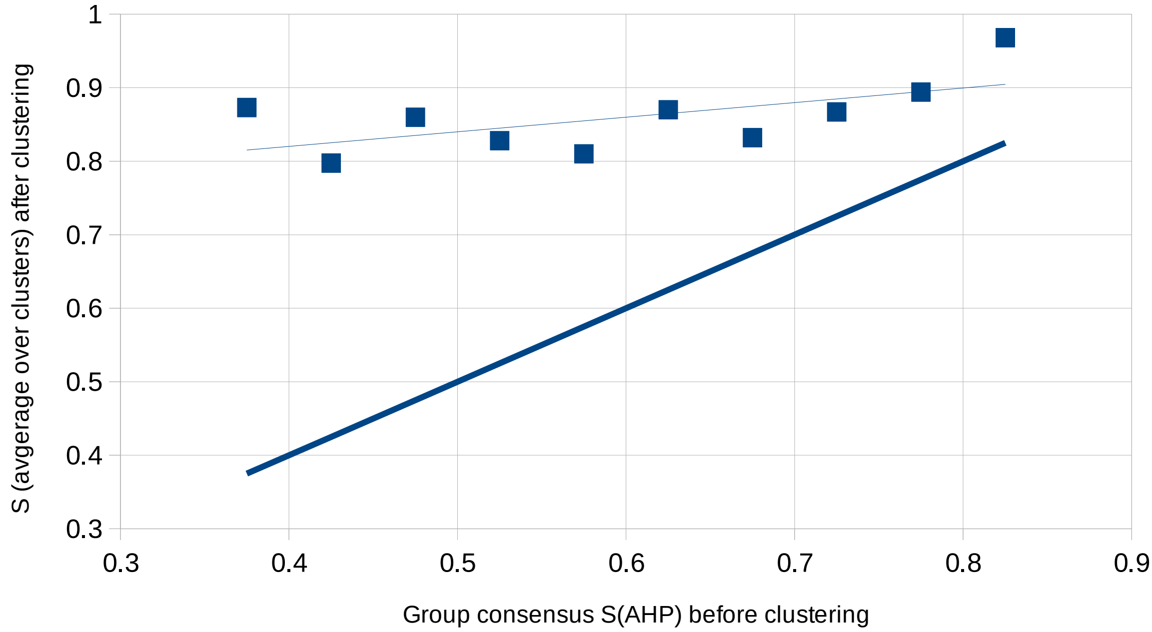

Figure 1. Improvement of consensus indicator S_{AHP} after clustering

Example 1

In the first example we look at a project node with three criteria judged by 13 participants. Priorities of the whole group is displayed in figure 2. The consolidated group result shows over 50 % weight for criterion-1, approx. 30 % for criterion-2 and 20 % for criterion-3. The group consensus is with 47 % very low.

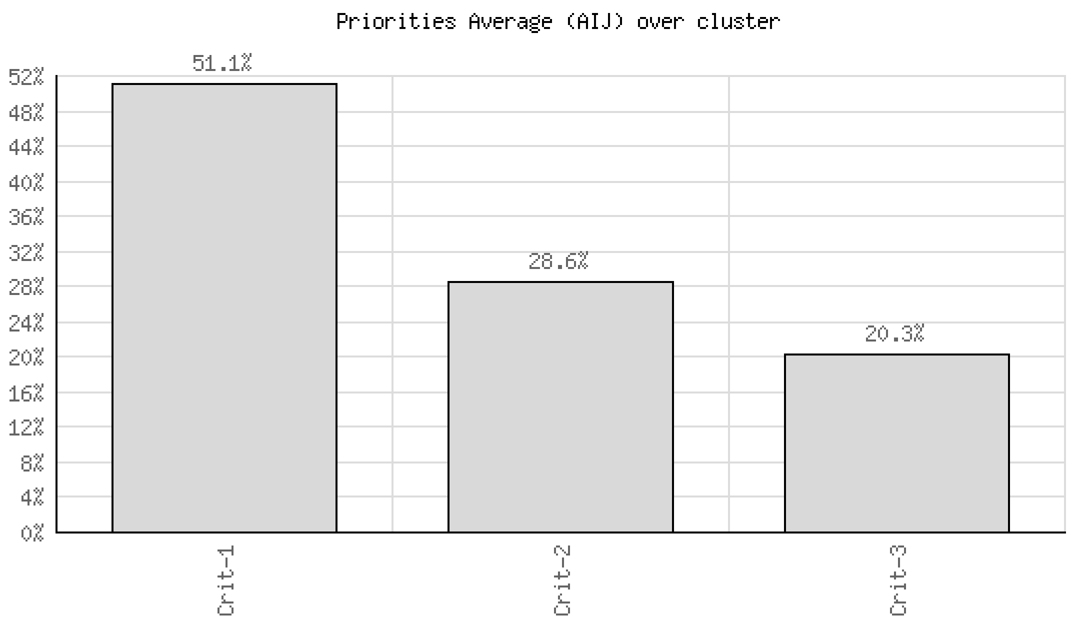

Figure 2. Priorities of the whole group for example 1.

The threshold table (table 2) shows the number of possible clusters and the number of dissociated participants as a function of the threshold value. The number of clusters is one or two, the number of dissociated participants increases at higher threshold values.

Table 2: Threshold table of example 1

Threshold	0.975	0.95	0.925	0.9	0.875	0.85	0.825	0.8	0.775	0.75	0.725	0.7
Clusters	2	2	2	2	2	**2**	2	2	2	2	1	1
Dissociated	6	4	2	1	1	**0**	0	0	0	0	2	1

Under the boundary conditions given before, the algorithm determines an optimal threshold value of 0.85, resulting in two clusters.

The first cluster consists of eight participants (1, 2, 4, 5, 9, 10, 11, and 13); the second cluster of five participants (3, 6, 7, 8, and 12). The consensus of cluster one is increased from 47 % (very low) to 90.1 % (very high), for cluster two from 47 % to 78.3 % (high). The clusters can also clearly be recognized as rectangles along the diagonal in the similarity matrix (Fig. 3).

0	2	1	13	5	9	4	11	10	3	6	8	7	12
2	100%	97%	87%	90%	99%	92%	86%	95%	81%	79%	55%	45%	42%
1	97%	100%	95%	98%	97%	99%	96%	99%	78%	67%	45%	40%	29%
13	87%	95%	100%	99%	85%	95%	98%	91%	85%	64%	50%	52%	31%
5	90%	98%	99%	100%	89%	99%	100%	96%	78%	60%	41%	41%	23%
9	99%	97%	85%	89%	100%	93%	86%	96%	75%	73%	48%	37%	34%
4	92%	99%	95%	99%	93%	100%	98%	99%	73%	58%	36%	33%	19%
11	86%	96%	98%	100%	86%	98%	100%	94%	76%	56%	38%	39%	20%
10	95%	99%	91%	96%	96%	99%	94%	100%	70%	60%	35%	29%	20%
3	81%	78%	85%	78%	75%	73%	76%	70%	100%	90%	86%	85%	71%
6	79%	67%	64%	60%	73%	58%	56%	60%	90%	100%	92%	79%	86%
8	55%	45%	50%	41%	48%	36%	38%	35%	86%	92%	100%	94%	96%
7	45%	40%	52%	41%	37%	33%	39%	29%	85%	79%	94%	100%	87%
12	42%	29%	31%	23%	34%	19%	20%	20%	71%	86%	96%	87%	100%

Figure 3. Clustered similarity matrix for the 13 participants in example 1.

The averaged priorities of each cluster are shown figure 4 and 5. We see that members of cluster one put their emphasis on criterion-1 with a weightage of 73 % (Fig. 4), whereas members of cluster two put their main emphasis on criterion-2 and -3 (62 % resp. 26 %, Fig. 5). It is a "split" decision with the majority of eight out of 13 (62 %) participants ranking criterion-1 first, and four out of 13 (31 %) ranking criterion-2 first.

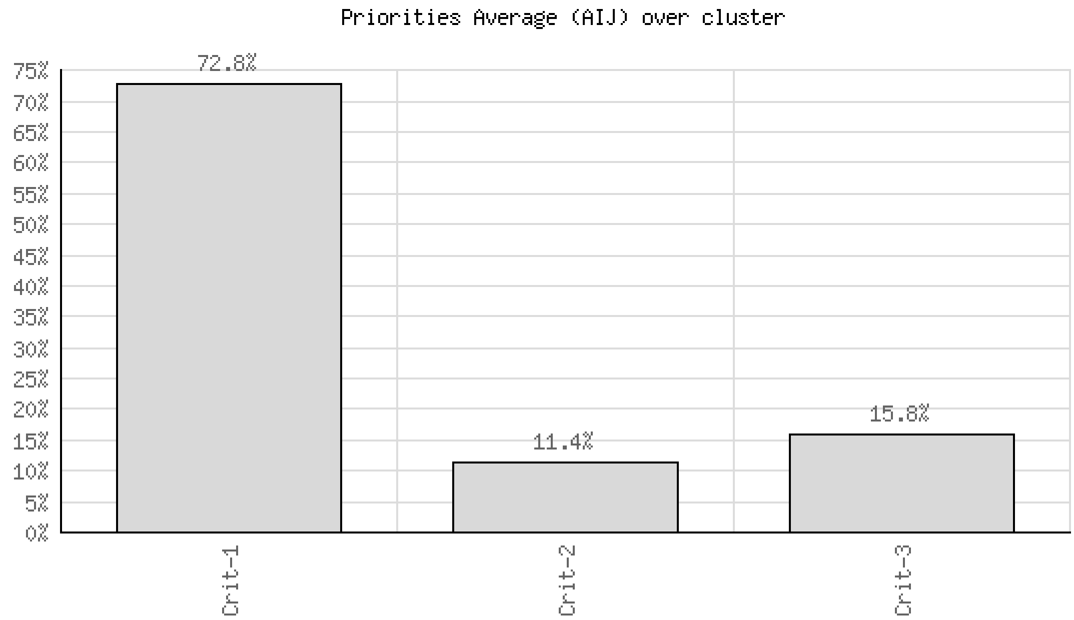

Figure 4. New priorities for the first cluster in example 1.

Figure 5: New priorities for the second cluster in example 1.

Example 2

In our second example we look at four criteria evaluated by a small group of seven participants. The group consensus of the whole group is with 40.4 % very low. Figure 6 shows the similarity matrix before clustering.

0	1	2	3	4	5	6	7
1	100%	48%	85%	50%	76%	85%	39%
2	48%	100%	40%	93%	46%	41%	95%
3	85%	40%	100%	55%	93%	100%	39%
4	50%	93%	55%	100%	63%	56%	94%
5	76%	46%	93%	63%	100%	95%	42%
6	85%	41%	100%	56%	95%	100%	39%
7	39%	95%	39%	94%	42%	39%	100%

Figure 6: Similarity matrix of example 2 before clustering.

The threshold table (table 3) shows two clusters up to a threshold of 0.95 with the minimum of $m_{cl} + m_{uc}$ at 0.85.

Table 3: Threshold table of example 2

Threshold	0.975	0.95	0.925	0.9	0.875	0.85	0.825	0.8	0.775	0.75	0.725	0.7
Clusters	1	2	2	2	2	**2**	2	2	2	2	2	2
Dissociated	5	2	1	1	1	**0**	0	0	0	0	0	0

Figure 7 shows the corresponding similarity matrix M for $th = 0.85$. We can see the two clusters along the diagonal. The first cluster contains four participants (1, 3, 5, and 6), the group consensus is significantly increased from 40.4 % (very low) to 83.4 % (high). The second cluster contains three participants (2, 4, and 7) with a group consensus of 91.8 % (very high). We now know that we have two subgroups with distinct preferences in this example.

0	6	5	3	1	4	2	7
6	100%	95%	100%	85%	56%	41%	39%
5	95%	100%	93%	76%	63%	46%	42%
3	100%	93%	100%	85%	55%	40%	39%
1	85%	76%	85%	100%	50%	48%	39%
4	56%	63%	55%	50%	100%	93%	94%
2	41%	46%	40%	48%	93%	100%	95%
7	39%	42%	39%	39%	94%	95%	100%

Figure 7: Similarity Matrix of the second example after clustering.

Looking at the averaged priorities for the two clusters (Fig. 8 and 9), we also recognize the different weight distribution. Cluster one emphasizes criterion-1 with 62 %, while cluster two emphasizes criterion-2 and 4 with 52 % resp. 33 %.

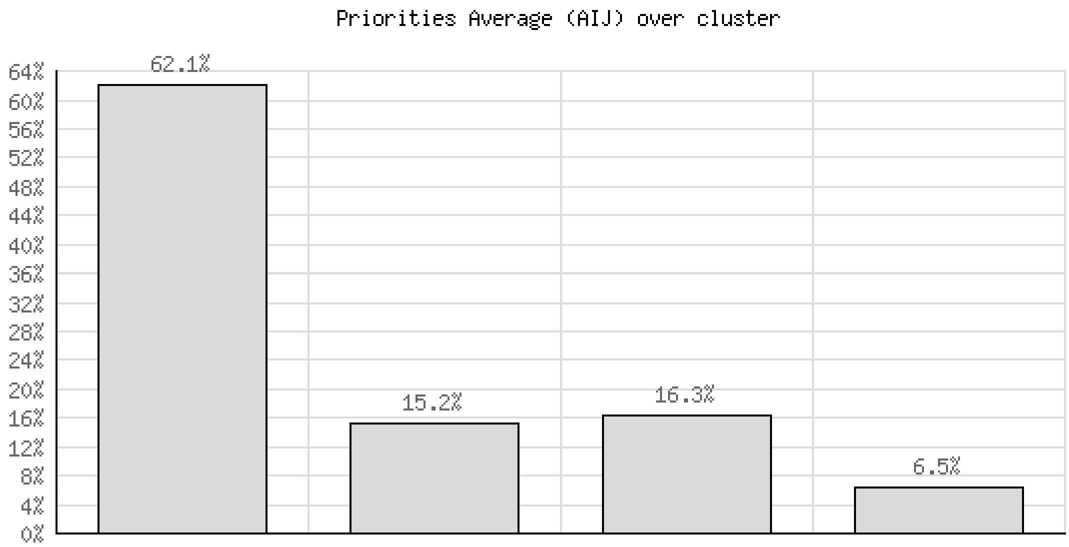

Figure 8: Weight distribution for the first cluster in example 2.

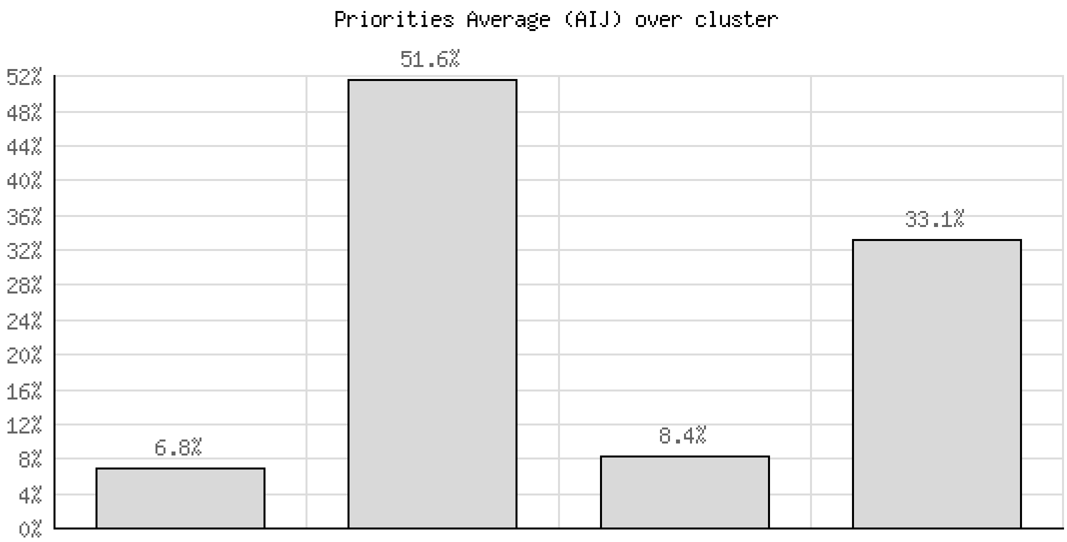

Figure 9: Weight distribution for the second cluster in example 2.

Again, the group is split into four, respective three members with significant different preferences on a set of four criteria.

5. Limitations

The actual implementation of the algorithm uses the arithmetic mean as aggregation function of weights, and group consensus of clusters is calculated based on AIP. This could be easily changed, for example, using individual judgments for the similarity matrix and the geometric mean as an aggregation function of individual judgments (AIJ)[25] for clustering.

The algorithm was developed with the intention to run online and allow users to get a quick insight into their group decision projects. Therefore, we looked at a simple and fast solution for clustering. The proposed clustering might not always be the optimal solution, there is still room to experiment with different boundary conditions to optimize the process. On the other hand, we have provided a manual input for the threshold values, and display the output of the similarity matrix for visualization, therefore users have the possibility to deviate from the proposed clustering and adapt the final clustering to their projects.

The similarity matrix can be displayed up to a size of 40 by 40 as full matrix showing elements values, and up to 150 by 150 showing colour coded elements without actual values. The program was tested to run with group sizes of approximately 800 participants, execution time will increase for larger sample sizes and an online execution will no longer be optimal.

6. Conclusion

Multi-criteria decision making support tools are helpful when making group decisions. While mathematically individual inputs always can be aggregated to yield a group result, it is important to analyse the outcome using a consensus indicator as a measure of agreement among group members. The author introduced a consensus indicator derived from Shannon entropy, which can be partitioned into two independent alpha and beta components. The partitioning allows to compare the similarity of priority distributions over categories between all pairs of group members and arrange them in a similarity matrix. A simple clustering algorithm was developed to identify potential smaller subgroups with higher consensus within the whole group. Using randomly selected samples of data resulting from AHP projects with group sizes ranging from small (10^1) to large (10^2) it was shown that for many of the projects the group could be divided into two or three smaller subgroups with a significant higher consensus and dissimilar judgments on specific criteria. This provides a good insight into the group results and could be used, for example, to initiate a further meeting between the subgroups to discuss the results and find a suitable compromise.

The cluster algorithm developed is actually not limited to group decision making; it is general in its application. Beta diversity as a measure of variation (similarity and overlap) between different samples of data distributions can also be used in the field of business analysis. The author used it, for example, to analyse similarity of markets, product and market diversification and to track the success of derived business actions.

The algorithm was implemented as a new function in the free, web based AHP Online System,[26] an open source software tool for educational and research purposes. The author wants to thank all users for their support and help with raw data on the platform for test and development.

References

1. B. Alfurhood, M. Silaghi, A Survey of Group Decision Making Methods and Evaluation Techniques, *Proceedings of the Thirty-First International Florida Artificial Intelligence Research Society Conference* (FLAIRS 2018) 168–171, ISBN 978-1-57735-796-4.
2. Özgür Kabak, Bilal Ervural, Multiple attribute group decision making: A generic conceptual framework and a classification scheme, *Knowledge-Based Systems* **123** (2017) 13–30.
3. L. Zadnik-Stirn, & P. Grošelj, Multiple criteria methods with focus on analytic hierarchy process and group decision making, *Croatian Operational Research Review* **1** (2010) 2–11.

4. E. Abel, L. Mikhailov, J. Keane, Group aggregation of pairwise comparisons using multiobjective optimisation, *Information Sciences*, **322** (2015) 257–275.
5. P. Grošelj, L. Z. Stirn, N. Ayrilmis, M. K. Kuzman, Comparison of some aggregation techniques using group Analytic Hierarchy Process. *Expert Systems with Applications* **42** (2015) 2198–2204, doi: 10.1016/j.eswa.2014.09.060.
6. R. Wan, M. L. A. Wan, Aggregation Methods in Group Decision Making: A Decade Survey. *Informatica* **41** (2017) 71–86.
7. Wen-Hsiang Wu, Chang-tzu Chiang, Chin-tsai Lin, Comparing the aggregation methods in the Analytic Hierarchy Process when uniform distribution. *WSEAS Transactions on Business And Economics*, **5**(3) (2008), 82–87.
8. E. Formana, K. Peniwati, Aggregating individual judgments and priorities with the analytic hierarchy process. *European Journal of Operational Research*, Volume **108**, Issue 1 (1998) 165–169.
9. D. K. Carmo, F. A. Marins, V. A. Salomon, C. H. Mello, On The Aggregation of Individual Priorities in Incomplete Hierarchies. *The International Symposium on the Analytic Hierarchy Process* (2013) doi: 10.13033/isahp.y2013.058.
10. S. Bennani, M. K. Idrissi, F. Nourredine, Y. B. Touimi. The homogeneity indicator of learners in project-based learning. *International Journal of Computer Science*, **11**(1) (2014) 188–194.
11. Y. Dong, G. Zhang, W.-C. Hong, Y. Xu, Consensus models for ahp group decision making under row geometric mean prioritization method. *Decision Support Systems* **49** (2010) 281–289.
12. K. D. Goepel, Implementing the Analytic Hierarchy Process as a standard method for Multi-Criteria Decision Making in corporate enterprises - A new AHP Excel template with multiple inputs. *Proceedings of the International Symposium on the Analytic Hierarchy Process for Multicriteria Decision Making* (2013), Kuala Lumpur, Malaysia.
13. E. K. Morris, T. Caruso, F. Buscot, M. Fischer, C. Hancock, T. S. Maier, T. Meiners, C. Müller, E. Obermaier, D. Prati, S. A. Socher, I. Sonnemann, N. Wäschke, T. Wubet, S. Wurst, & M. C. Rillig, Choosing and using diversity indices: insights for ecological applications from the German Biodiversity Exploratories. *Ecology and evolution* **4**(18) (2014) 3514–3524 doi: 10.1002/ece3.1155.
14. C. Shannon, A mathematical theory of communication. *Bell System Tech. J.* **27** (1949) 379-423, 623-656.
15. Hill, M. O., Diversity and evenness: a unifying notation and its consequences. *Ecology* **54** (1973) 427–431, doi: 10.2307/1934352.
16. L. Jost, Entropy and diversity. *Oikos* **113** (2006) 363–375, doi: 10.1111/j.2006.0030-1299.14714.x.
17. Anne Chao, Chun-Huo Chiu, L. Jost, Phylogenetic Diversity Measures and Their Decomposition: A Framework Based on Hill Numbers, *Biodiversity Conservation and Phylogenetic Systematics*, Springer International Publishing (2016) 141–172 doi: 10.1007/978-3-319-22461-9_8, ISBN 9783319224602.
18. W. J. Tastle, & M. J. Wierman, Consensus and dissention: a measure of ordinal dispersion. *International Journal of Approximate Reasoning* **45** (2007) 531–545.
19. R. H. Whittaker, Evolution and measurement of species diversity, *Taxon* **12** (1972) 213–251.
20. L. Jost, Partitioning diversity into independent alpha and beta components. *Ecology* **88** (2007) 2427–2439, doi: 10.1890/06-1736.1).

21. R. H. MacArthur, Patterns of species diversity. *Biol. Rev.* **40** (1965) 510 –533, doi: 10.1111/j.1469-185X.1965.tb00815.x.
22. T. L. Saaty, The Analytic Hierarchy Process: Planning, priority setting, resource allocation. McGraw-Hill 1980, doi: 10.1016/0377-2217(82)90022-4.
23. K. D. Goepel, Implementation of an Online Software Tool for the Analytic Hierarchy Process (AHP-OS). *International Journal of the Analytic Hierarchy Process*, Vol. **10** Issue 3 (2018) 469–487, doi: 10.13033/ijahp.v10i3.590.
24. PHP hypertext preprocessor (2022), https://www.php.net/
25. W. Ossadnik, S. Schinke, & R. H. Kaspar, Group aggregation techniques for analytic hierarchy process and analytic network process: a comparative analysis. *Group Decision and Negotiation* **25**(2) (2015), 421–457, doi: 10.1007/s10726-015-9448-4.
26. K. D. Goepel, AHP Online System – AHP-OS (2022), https://bpmsg.com/ahp/