Conjoint Analysis
Basic Principle

Conjoint Analysis

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Conjoint Analysis
Basic Principle

Conjoint Analysis

The presentation explains the principle, using a simple example. It shows, how to calculate the part-worth utilities and how to derive the relative preferences from individual attributes from there. A full factorial and a fractional factorial design is used. An Excel template for this example is available from the author.

Conjoint analysis or **Stated preference analysis** is a statistical technique that originated in mathematical psychology.

Conjoint Analysis

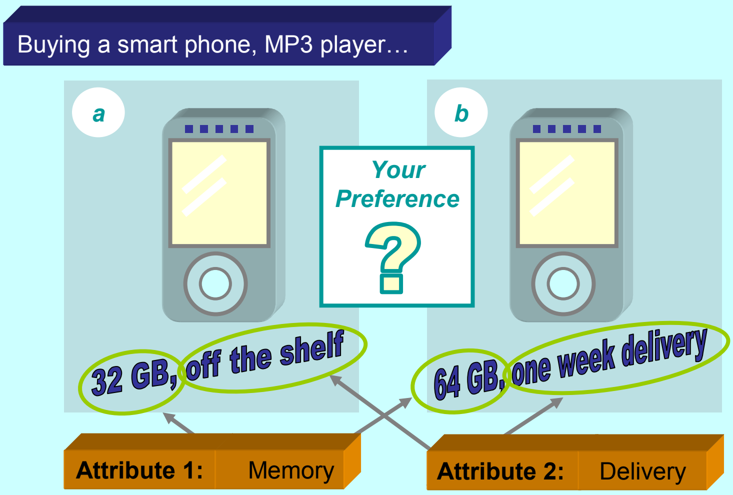
Keywords

conjoint analysis, stated preference analysis, linear regression, product management, marketing, part-worth, utilities, relative preference, statistics, analytic hierarchy process, AHP

Conjoint Analysis
Basic Principle

Today it is used in many of the social sciences and applied sciences including

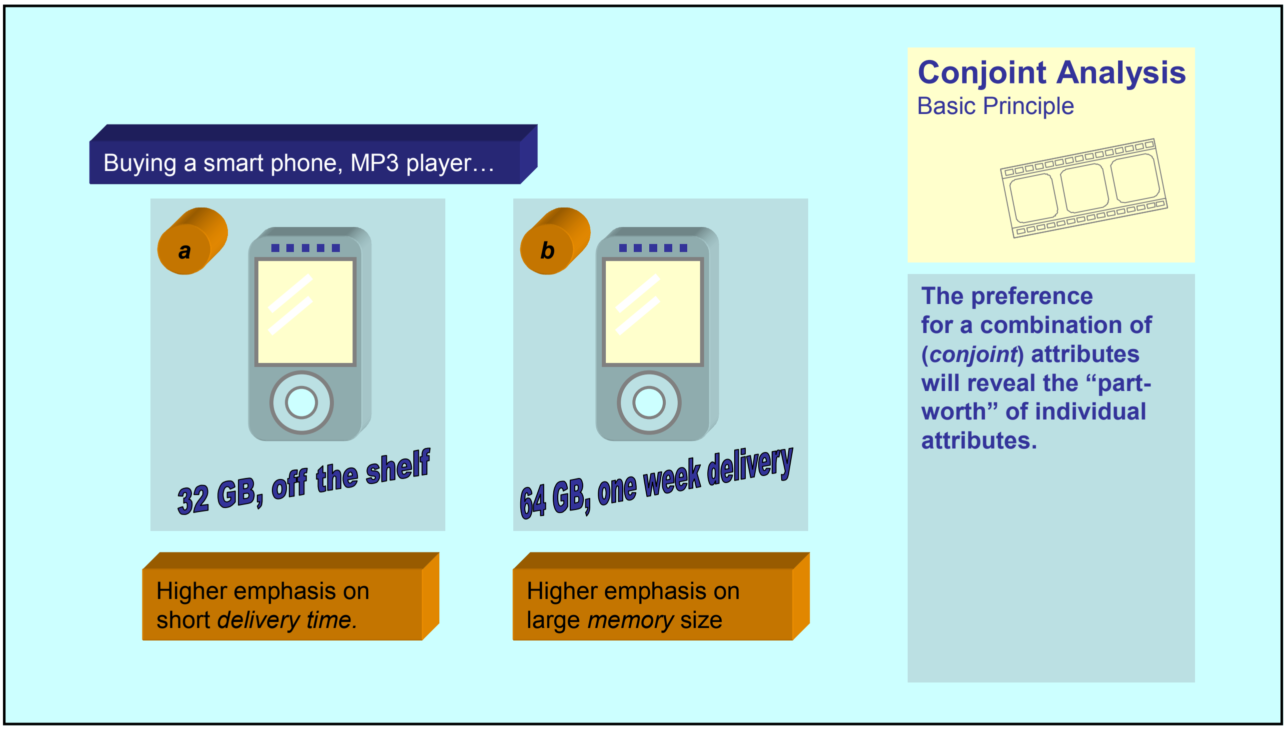
- Marketing,
- Product management,
- Operations research.

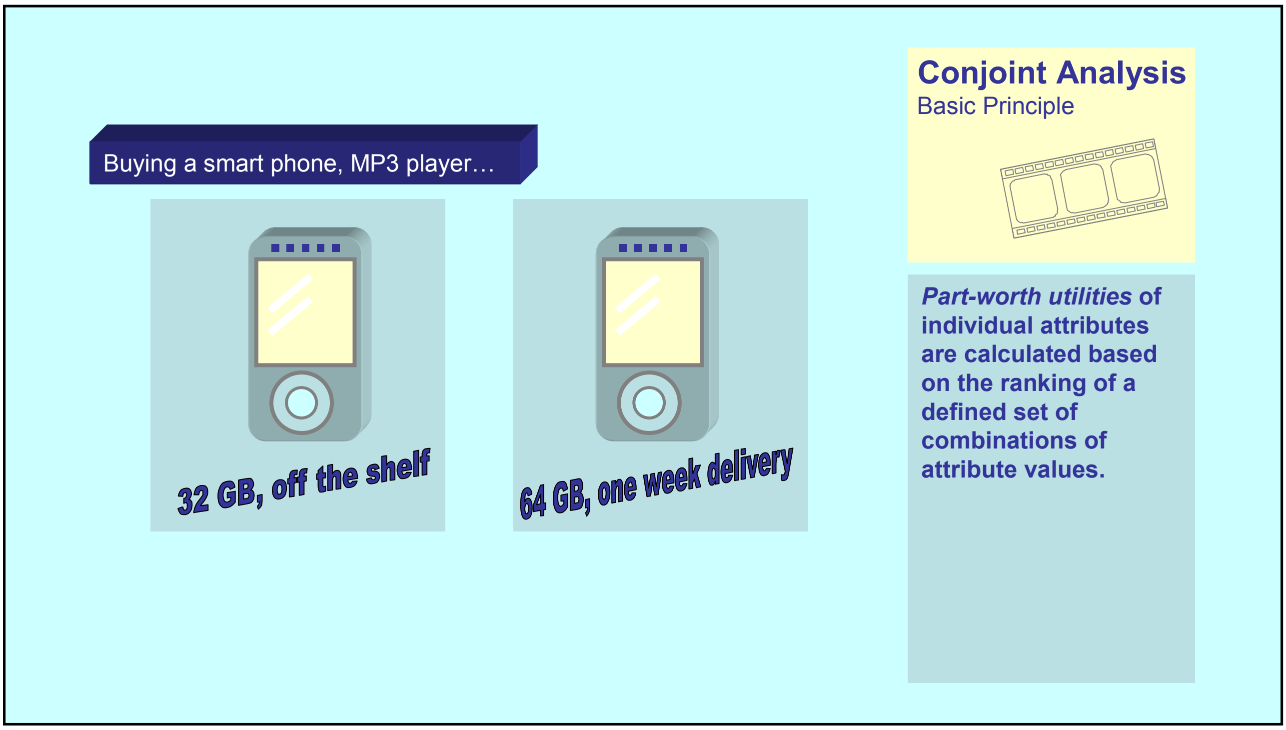


Conjoint Analysis
Basic Principle

The preference for a combination of (*conjoint*) attributes will reveal the "part-worth" of individual attributes.

Attribute 1: Memory
Attribute 2: Delivery


Conjoint Analysis
Basic Principle
Buying a smart phone, MP3 player…
a
b
32 GB, off the shelf
64 GB, one week delivery
Higher emphasis on short *delivery time*.
Higher emphasis on large *memory* size
The preference for a combination of (*conjoint*) attributes will reveal the “part-worth” of individual attributes.


Conjoint Analysis
Basic Principle
Buying a smart phone, MP3 player…
32 GB, off the shelf
64 GB, one week delivery
Part-worth utilities of individual attributes are calculated based on the ranking of a defined set of combinations of attribute values.

Conjoint Analysis
Basic Principle

Buying a smart phone, MP3 player…

Attributes:

Attribute	Levels
Color	green, red
Memory	16 MB, 64 MB
Delivery	1 day, 1 week

Models:

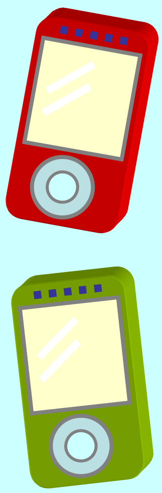
1. Green, 16 MB, 1 week
2. Red, 16 MB, 1 week
3. Green, 64 MB, 1 week
4. Red, 64 MB, 1 week
5. Green, 16 MB, 1 day
6. Red, 16 MB, 1 day
7. Green, 64 MB, 1 day
8. Red, 64 MB, 1 day

Example
Buying a smart phone

Attribute values are coded with -1 and +1
Attribute 1: Color
Attribute 2: Memory
Attribute 2: Delivery

Conjoint Analysis
Basic Principle

Sequence	"Conjoint" Attributes	Color	Memory	Delivery
1	Green, 16 MB, 1 week	-1	-1	-1
2	Red, 16 MB, 1 week	1	-1	-1
3	Green, 64 MB, 1 week	-1	1	-1
4	Red, 64 MB, 1 week	1	1	-1
5	Green, 16 MB, 1 day	-1	-1	1
6	Red, 16 MB, 1 day	1	-1	1
7	Green, 64 MB, 1 day	-1	1	1
8	Red, 64 MB, 1 day	1	1	1



Example
Buying a smart phone

Attribute values are coded with -1 and +1
Attribute 1: Color
Attribute 2: Memory
Attribute 2: Delivery

Conjoint Analysis
Basic Principle

Design Matrix

k attributes: 2^k possible combinations

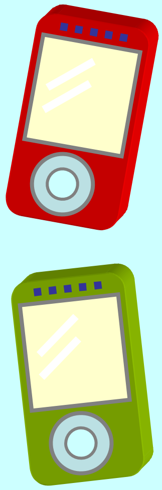
Full factorial Design

Full factorial design

k attributes:
2^k combinations

Design Matrix

-1	-1	-1
1	-1	-1
-1	1	-1
1	1	-1
-1	-1	1
1	-1	1
-1	1	1
1	1	1



Conjoint Analysis
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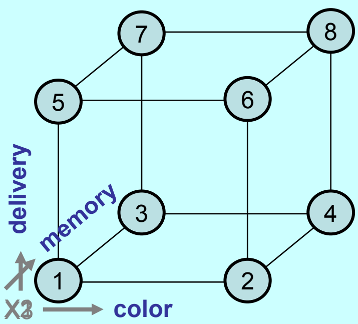
Graphical Representation of Combinations

Full factorial Design

X1: Color = (+1,-1)
X2: Memory = (+1,-1)
X3: Delivery = (+1,-1)



	X_1	X_2	X_3
1	-1	-1	-1
2	1	-1	-1
3	-1	1	-1
4	1	1	-1
5	-1	-1	1
6	1	-1	1
7	-1	1	1
8	1	1	1

Conjoint Analysis
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Sequence	"Conjoint" Attributes	X_1	X_2	X_3	Rank
1	Green, 16 MB, 1 week	-1	-1	-1	8
2	Red, 16 MB, 1 week	1	-1	-1	7
3	Green, 64 MB, 1 week	-1	1	-1	4
4	Red, 64 MB, 1 week	1	1	-1	3
5	Green, 16 MB, 1 day	-1	-1	1	6
6	Red, 16 MB, 1 day	1	-1	1	5
7	Green, 64 MB, 1 day	-1	1	1	2
8	Red, 64 MB, 1 day	1	1	1	1

Ranking of combinations

X1: Color = (+1,-1)
X2: Memory = (+1,-1)
X3: Delivery = (+1,-1)

Conjoint Analysis
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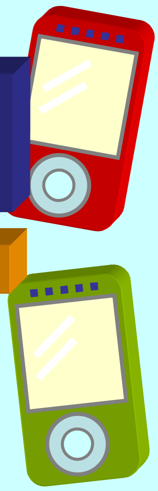
Linear model function with part-worth utilities

Ranking = part-worth of attribute 1 * attribute 1 level
+ part-worth of attribute 2 * attribute 2 level
+ part-worth of attribute 3 * attribute 3 level
+ baseline preference

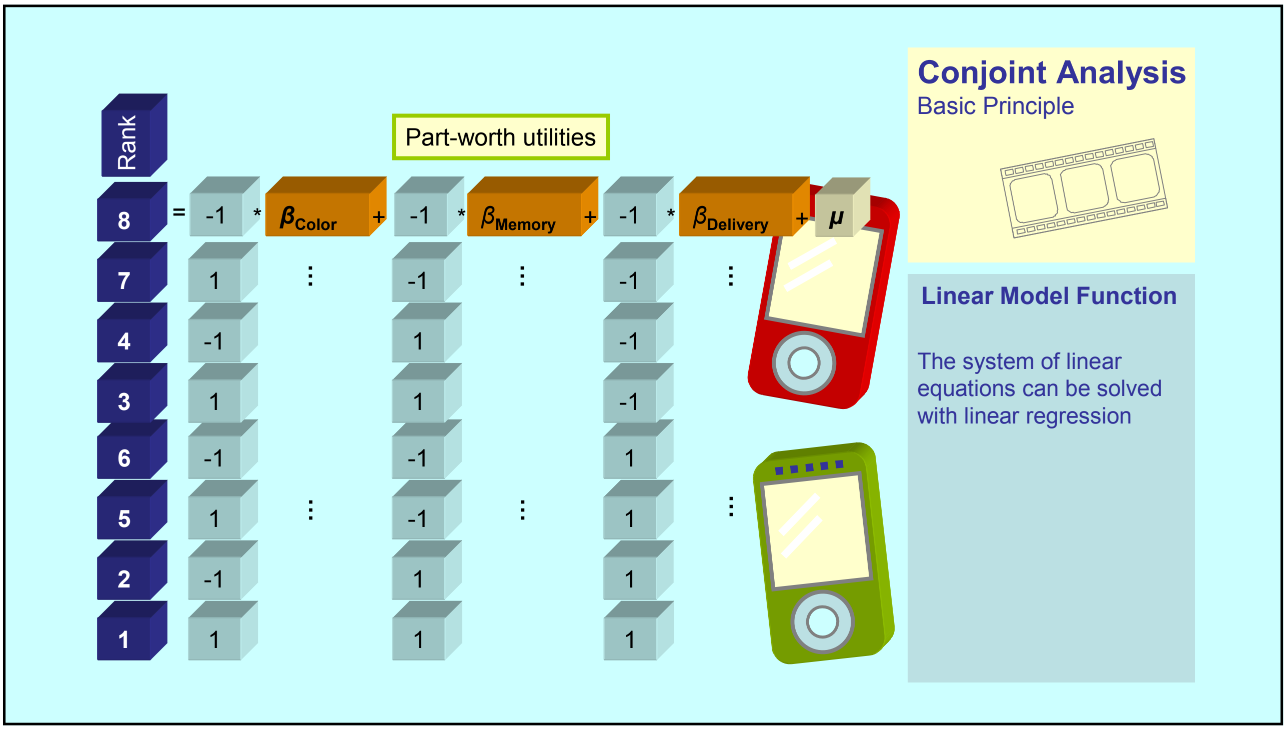
$$Y = \beta_{color} * X_1 + \beta_{memory} * X_2 + \beta_{delivery} * X_3 + \mu$$

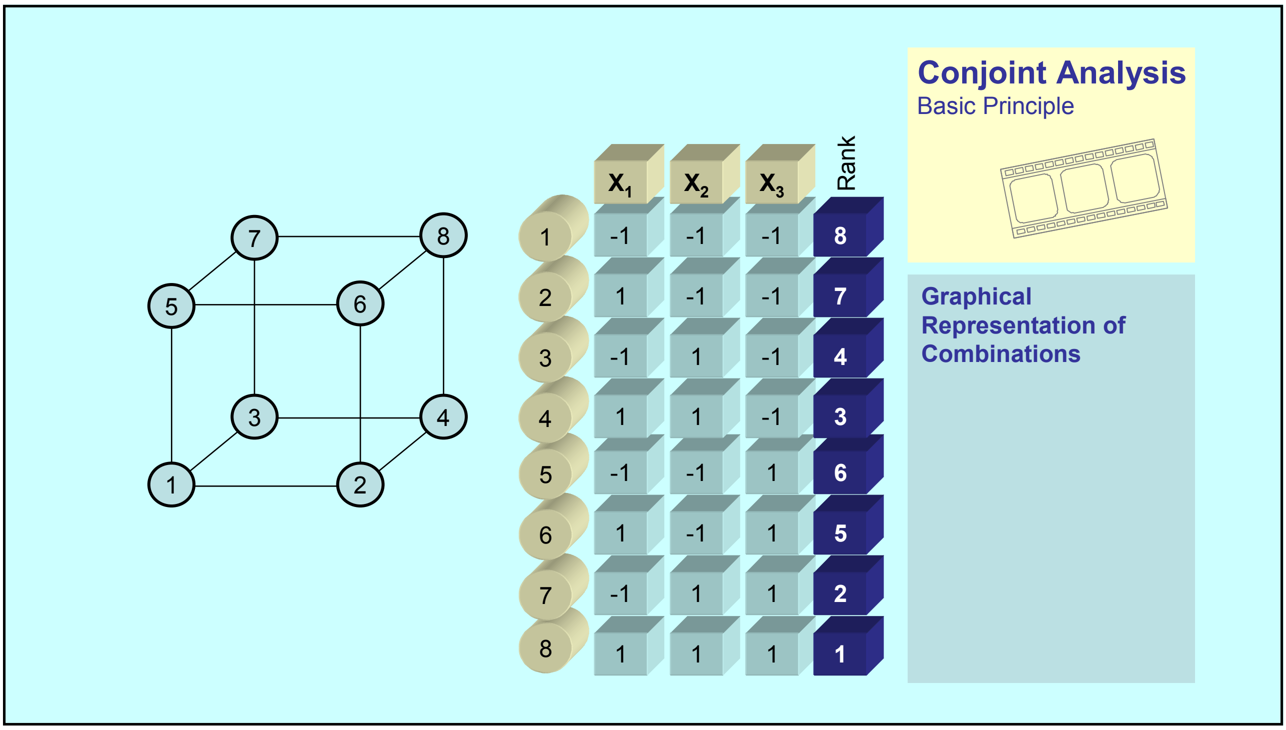


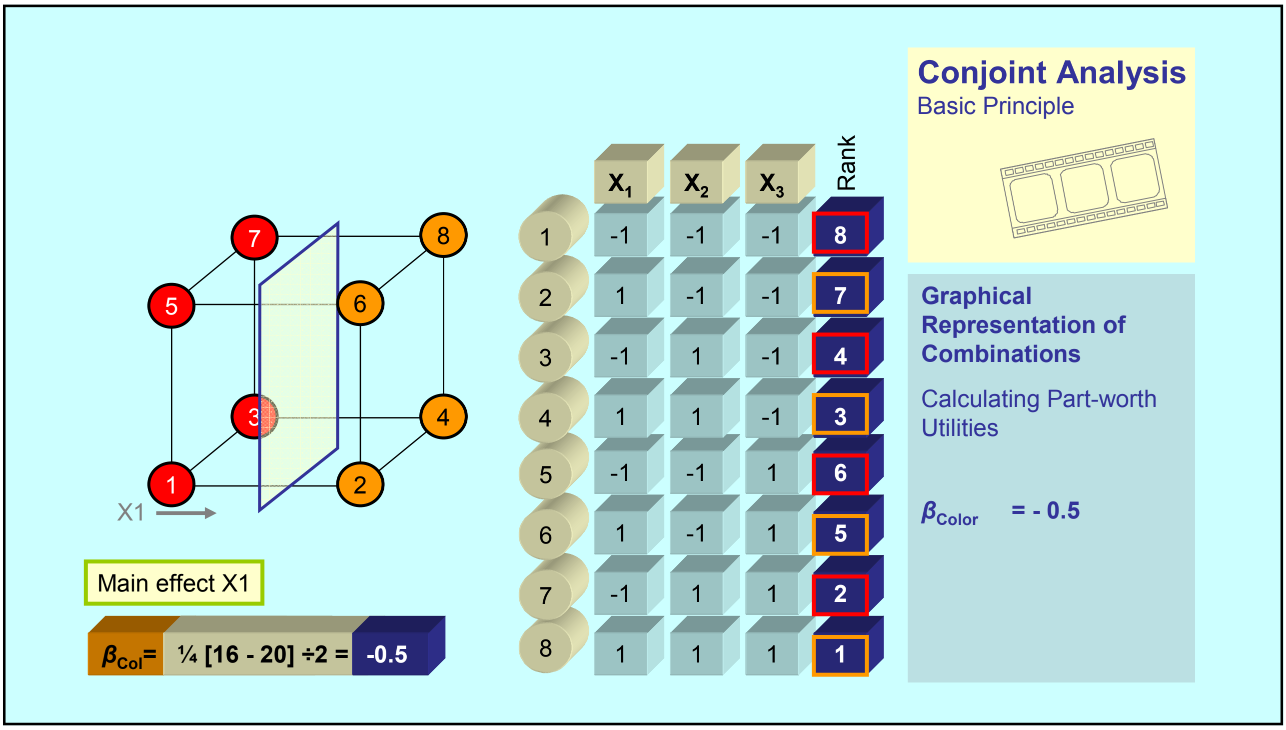
Linear Model Function

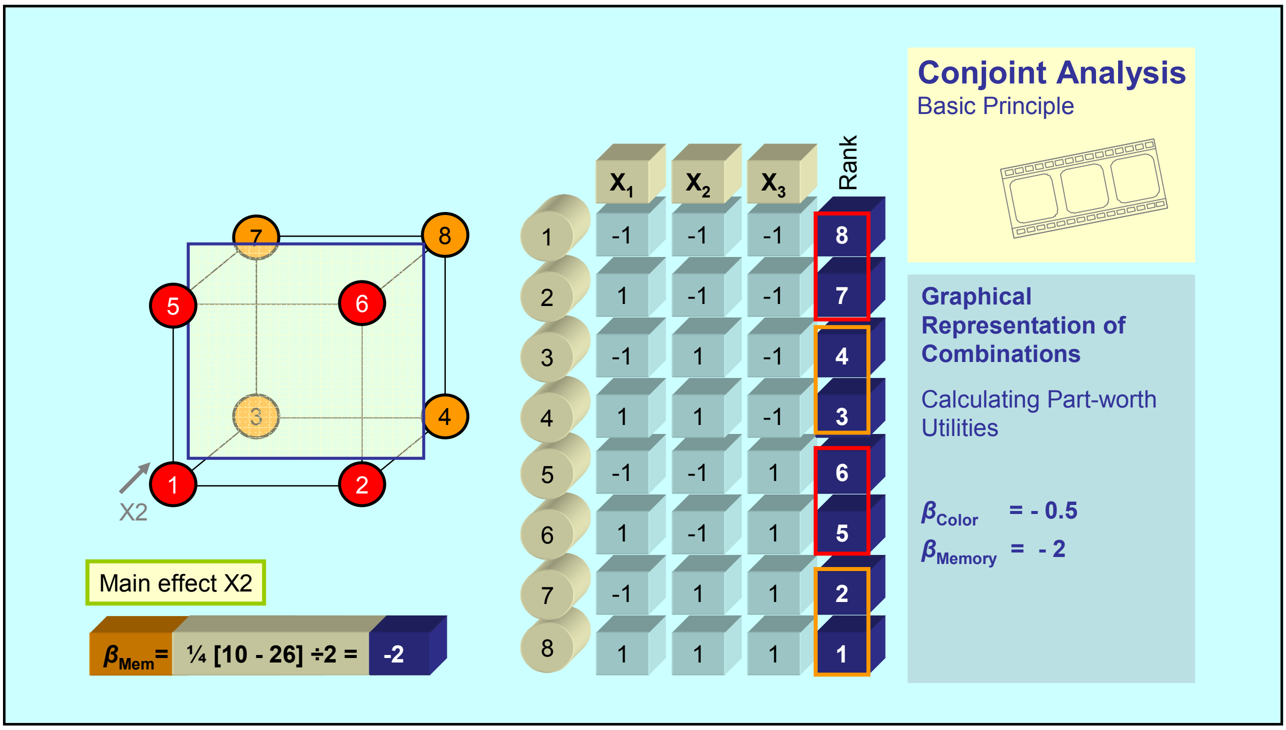
The system of linear equations can be solved with linear regression

X1: Color = (+1,-1)
X2: Memory = (+1,-1)
X3: Delivery = (+1,-1)


Conjoint Analysis
Basic Principle
Linear Model Function
The system of linear equations can be solved with linear regression
Part-worth utilities
Rank
βColor
βMemory
βDelivery
μ


Conjoint Analysis
Basic Principle
Graphical Representation of Combinations
X1
X2
X3
Rank


Conjoint Analysis
Basic Principle
Graphical Representation of Combinations
Calculating Part-worth Utilities
βColor = - 0.5
X1
Main effect X1
βCol= ¼ [16 - 20] ÷2 = -0.5
X1 X2 X3 Rank
1 -1 -1 -1 8
2 1 -1 -1 7
3 -1 1 -1 4
4 1 1 -1 3
5 -1 -1 1 6
6 1 -1 1 5
7 -1 1 1 2
8 1 1 1 1

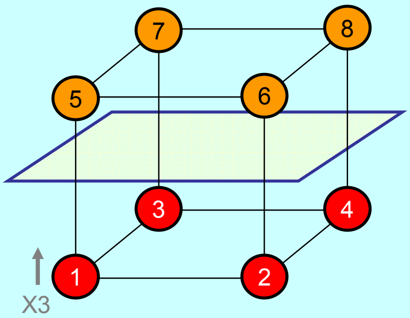

Conjoint Analysis
Basic Principle
Graphical Representation of Combinations
Calculating Part-worth Utilities
βColor = - 0.5
βMemory = - 2
X2
Main effect X2
βMem= ¼ [10 - 26] ÷2 = -2
X1 X2 X3 Rank
1 -1 -1 -1 8
2 1 -1 -1 7
3 -1 1 -1 4
4 1 1 -1 3
5 -1 -1 1 6
6 1 -1 1 5
7 -1 1 1 2
8 1 1 1 1

Conjoint Analysis
Basic Principle

Graphical Representation of Combinations

Calculating Part-worth Utilities

β_{Color} = - 0.5
β_{Memory} = - 2
$\beta_{Delivery}$ = - 1



Main effect X3

β_{Del}= ¼ [14 - 22] ÷2 = -1

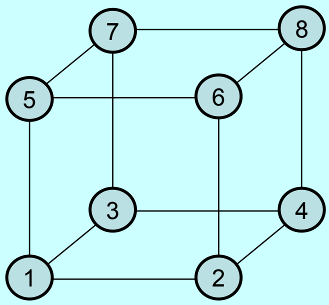
	X_1	X_2	X_3	Rank
1	-1	-1	-1	8
2	1	-1	-1	7
3	-1	1	-1	4
4	1	1	-1	3
5	-1	-1	1	6
6	1	-1	1	5
7	-1	1	1	2
8	1	1	1	1

Conjoint Analysis
Basic Principle

Graphical Representation of Combinations

Calculating Part-worth Utilities

β_{Color} = - 0.5
β_{Memory} = - 2
$\beta_{Delivery}$ = - 1

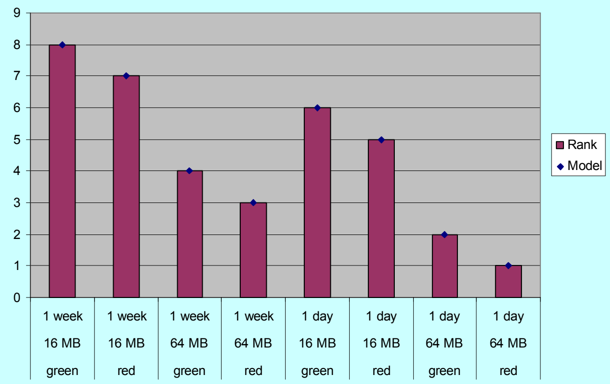


Part-worth utilities

β_{Color} = -0.5
β_{Mem} = -2
β_{Del} = -1

Conjoint Analysis
Basic Principle

Actual ranking and description with linear model function



Ranking and Model Function

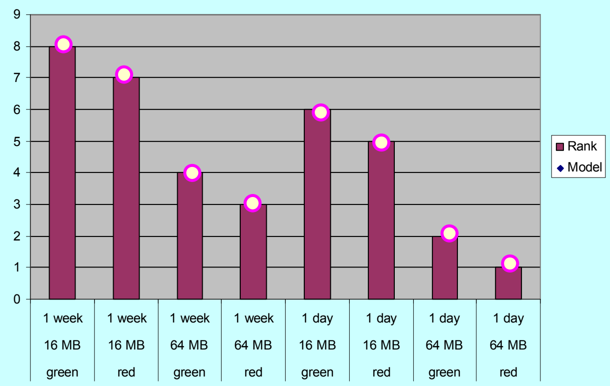
β_{Color} = - 0.5

β_{Memory} = - 2

$\beta_{Delivery}$ = - 1

Conjoint Analysis
Basic Principle

Actual ranking and description with linear model function



$$Y = 4.5 \ -0.5\, X_{color} \ -2\, X_{Memory} \ -1\, X_{Delivery}$$

Ranking and Model Function

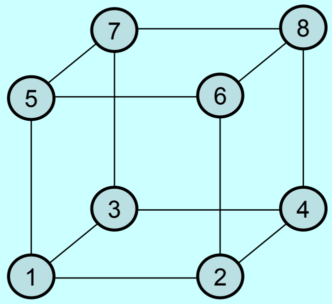
β_{Color} = - 0.5

β_{Memory} = - 2

$\beta_{Delivery}$ = - 1

Conjoint Analysis
Basic Principle

Calculating relative preferences



Variations for $X_i=\pm1$

$\pm1 \cdot \beta_{Col}$	=	**±0.5 = 1**
$\pm1 \cdot \beta_{Mem}$	=	**±2 = 4**
$\pm1 \cdot \beta_{Del}$	=	**±1 = 2**
		7

Color	1÷7 =	14%
Memory	4÷7 =	57%
Delivery	2÷7 =	29%

Conjoint Analysis
Basic Principle

Conjoint Analysis & Analytic Hierarchy Process AHP

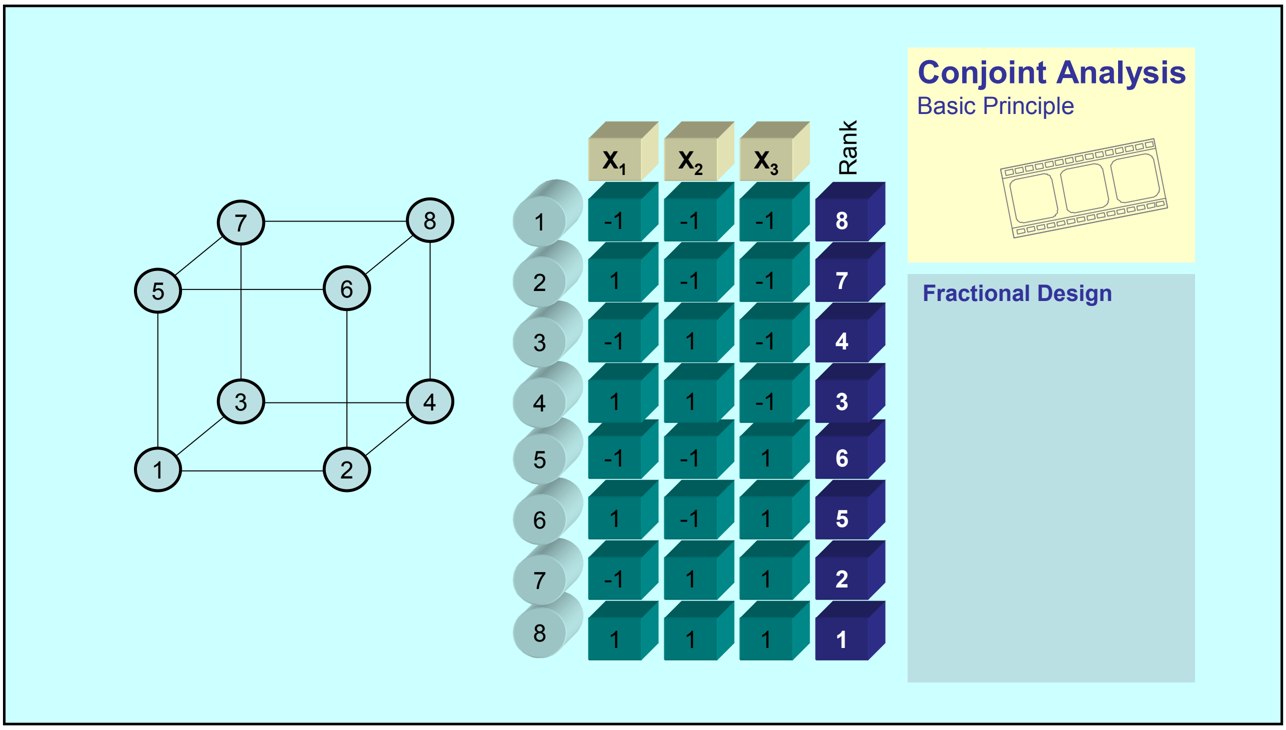
Conjoint Analysis	AHP
Attributes	**Criteria**, Sub-criteria - comparison
Part-worth Utilities	**Weights**: Principal Eigenvector
Levels	Ratio **Scale**, relative Scale
Ranking	**Evaluation of Alternatives**

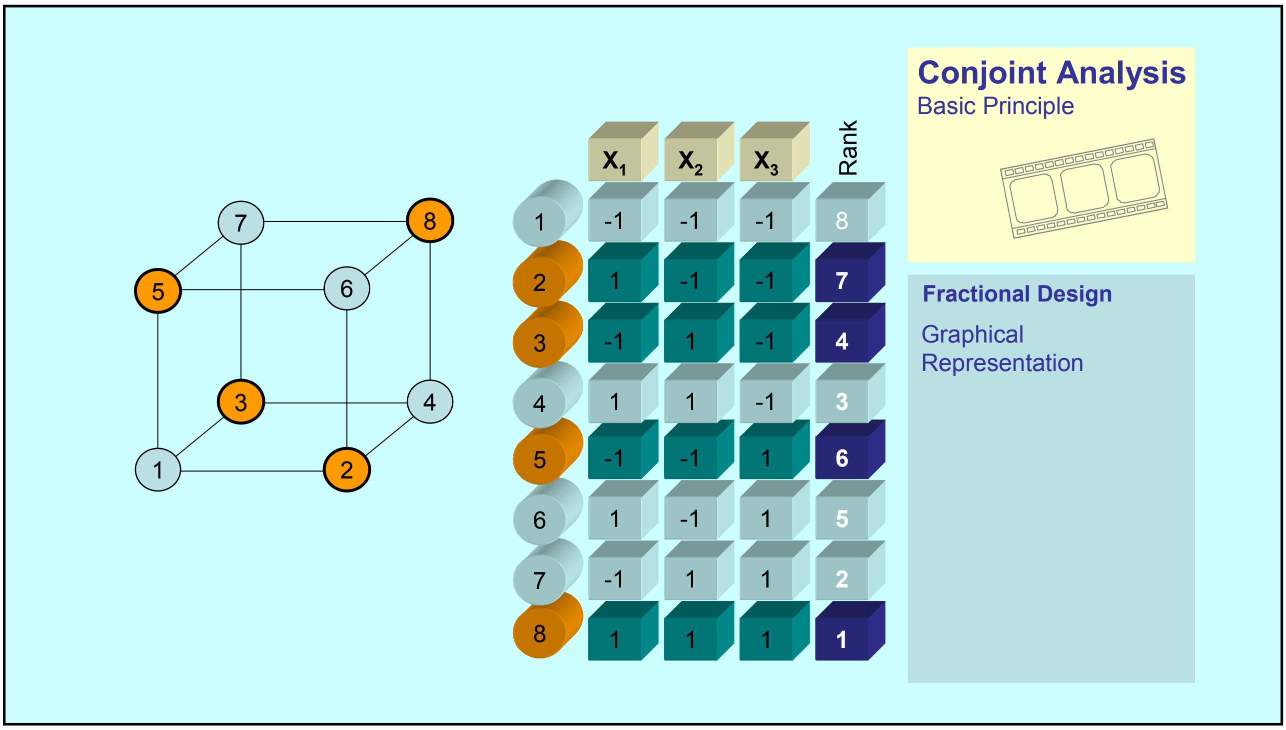
2^k Combinations

k=4: 16 possible combinations

$\frac{k^2-k}{2}$ Comparisons

k=4: 6 pair-wise comparisons


Conjoint Analysis
Basic Principle
Fractional Design
X1 X2 X3 Rank
1 -1 -1 -1 8
2 1 -1 -1 7
3 -1 1 -1 4
4 1 1 -1 3
5 -1 -1 1 6
6 1 -1 1 5
7 -1 1 1 2
8 1 1 1 1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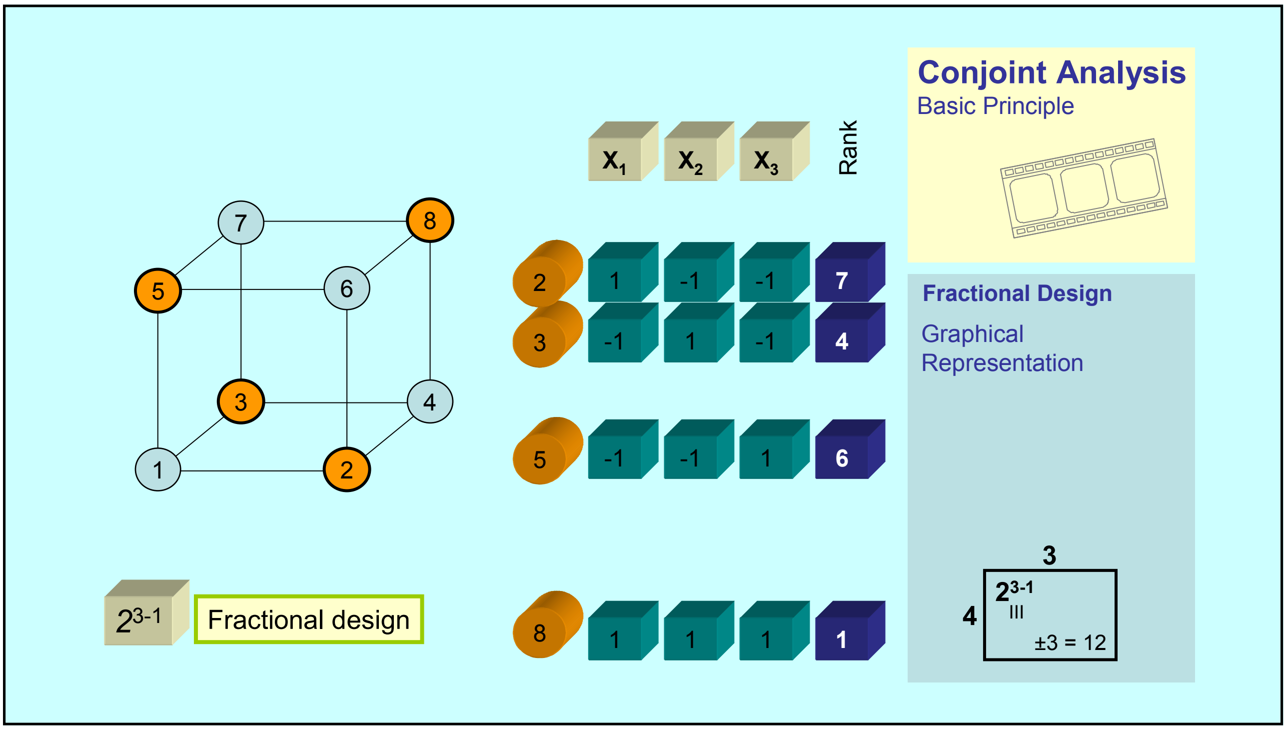
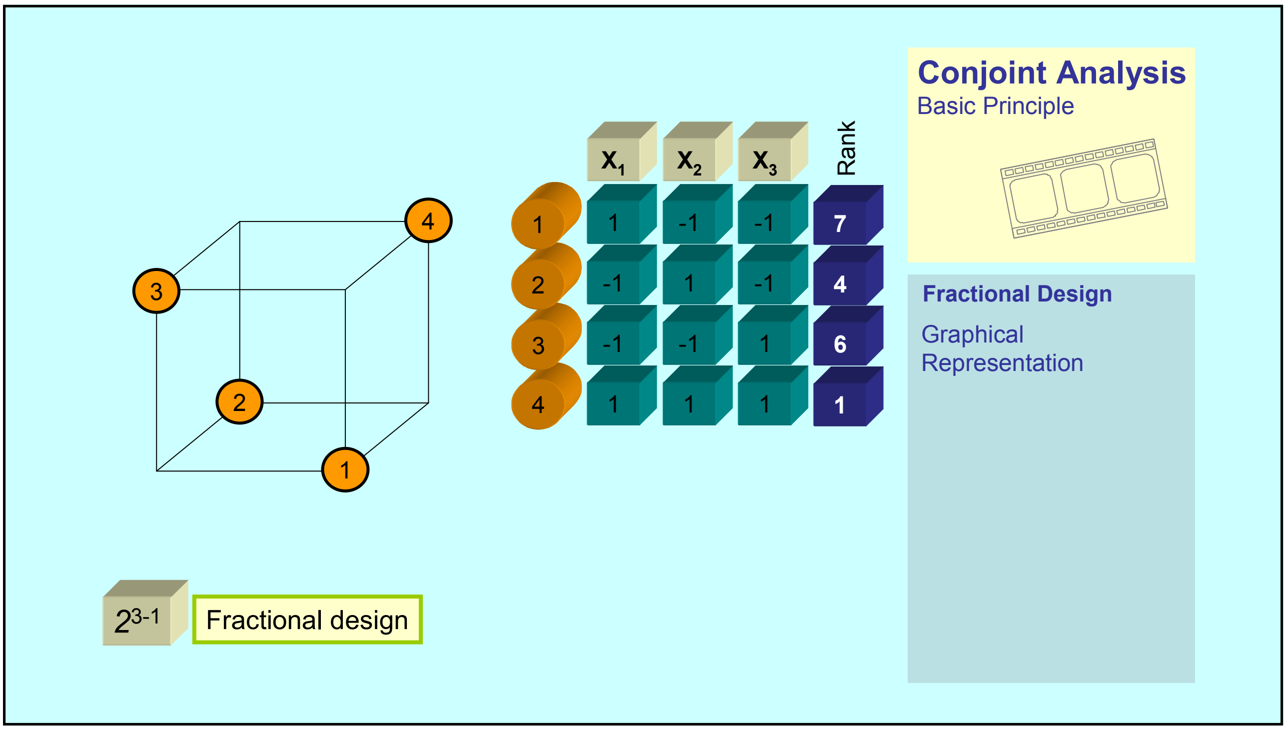
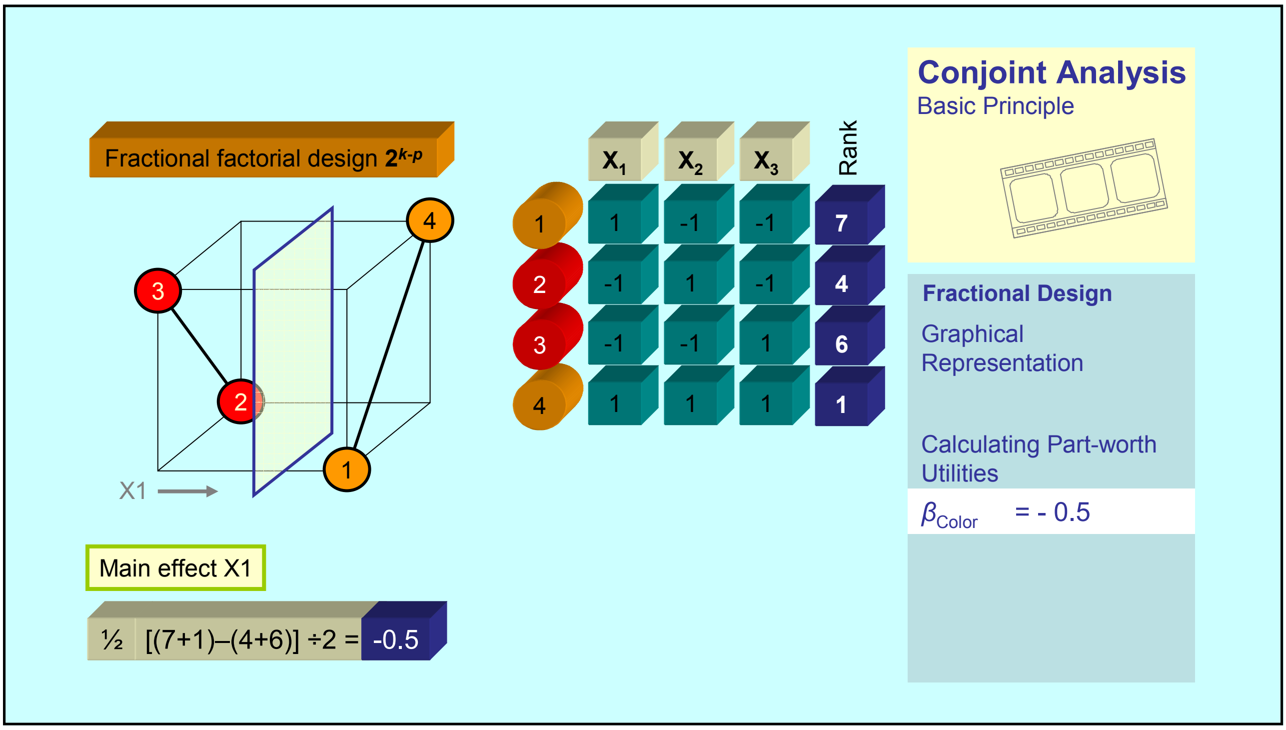
X1 X2 X3 Rank
1 -1 -1 -1 8
2 1 -1 -1 7
3 -1 1 -1 4
4 1 1 -1 3
5 -1 -1 1 6
6 1 -1 1 5
7 -1 1 1 2
8 1 1 1 1

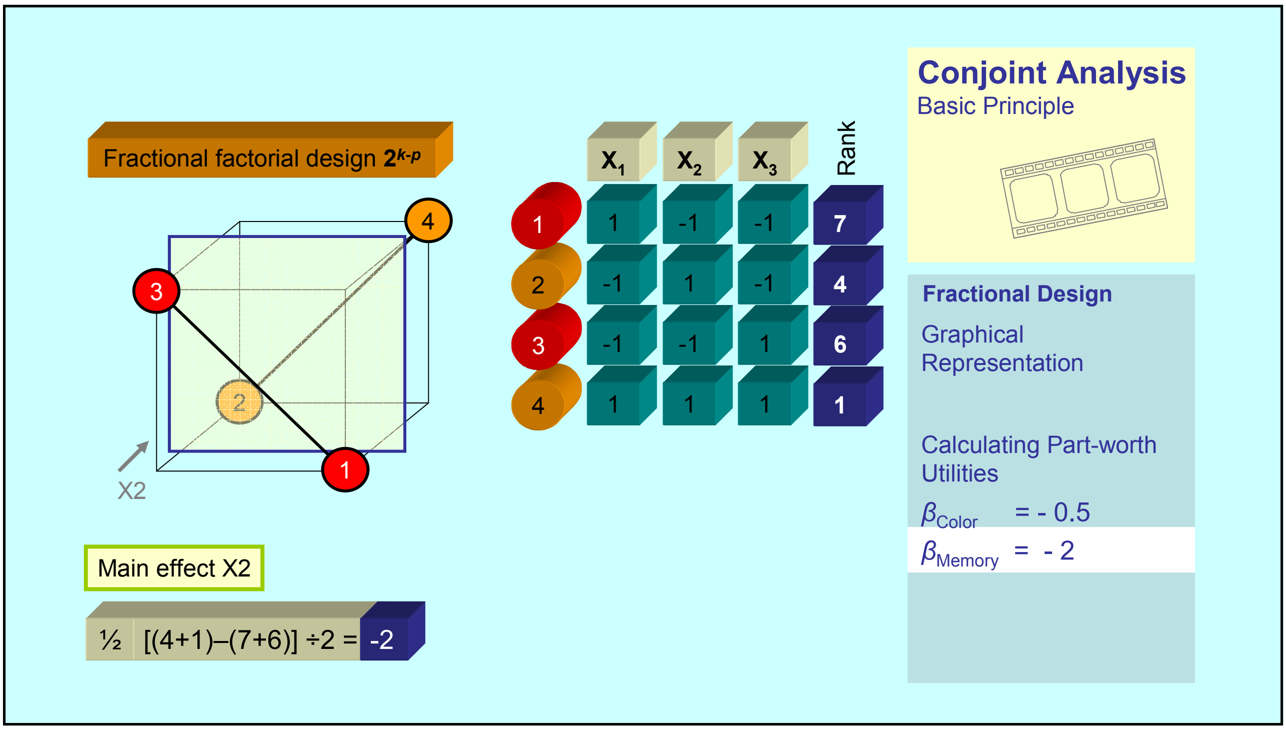


Conjoint Analysis
Basic Principle
Fractional Design
Graphical Representation
X1 X2 X3 Rank
2 1 -1 -1 7
3 -1 1 -1 4
5 -1 -1 1 6
8 1 1 1 1
7 8 5 6 3 4 1 2
2^3-1 Fractional design
3
4
2^3-1 III
±3 = 12

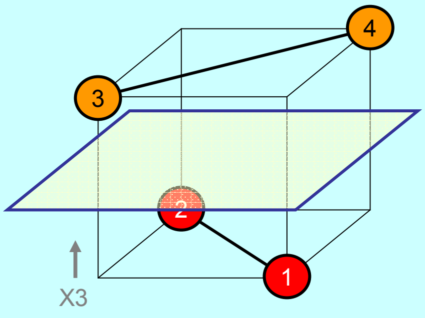


Conjoint Analysis
Basic Principle
Fractional Design
Graphical Representation
X1 X2 X3 Rank
1 1 -1 -1 7
2 -1 1 -1 4
3 -1 -1 1 6
4 1 1 1 1
4 3 2 1
2^3-1 Fractional design


Conjoint Analysis
Basic Principle
Fractional Design
Graphical Representation
Calculating Part-worth Utilities
β Color = - 0.5
Fractional factorial design 2k-p
X1
X2
X3
Rank
1
-1
-1
7
-1
1
-1
4
-1
-1
1
6
1
1
1
1
X1
Main effect X1
½ [(7+1)–(4+6)] ÷2 = -0.5


Conjoint Analysis
Basic Principle
Fractional Design
Graphical Representation
Calculating Part-worth Utilities
β Color = - 0.5
β Memory = - 2
Fractional factorial design 2k-p
X1
X2
X3
Rank
1
-1
-1
7
-1
1
-1
4
-1
-1
1
6
1
1
1
1
X2
Main effect X2
½ [(4+1)–(7+6)] ÷2 = -2

Fractional factorial design 2^{k-p}



	X_1	X_2	X_3	Rank
1	1	-1	-1	7
2	-1	1	-1	4
3	-1	-1	1	6
4	1	1	1	1

Main effect X3

½ [(6+1)–(7+4)] ÷2 = -1

Conjoint Analysis
Basic Principle

Fractional Design

Graphical Representation

Calculating Part-worth Utilities

β_{Color} = - 0.5

β_{Memory} = - 2

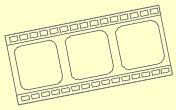
$\beta_{Delivery}$ = - 1

Conjoint Analysis
Basic Principle

Fractional Design

Using a fractional factorial design the number of attribute combinations can be reduced.

Conjoint Analysis
Basic Principle



Simple conjoint analysis can be done with linear regression, but more sophisticated statistical models and solutions are available.