Analytic Hierarchy Process

- Multi Criteria decision making method
- Originally developed by Prof. Thomas L. Saaty

Klaus Goepel, Mar. 2010

For more visit http://bpmsg.com
Analytic Hierarchy Process (AHP)
Deriving ratio scales from paired comparisons.

Allows some small inconsistency in judgment.

Input:
- Actual measurement
- Subjective opinion

Output:
- Ratio scales
- Consistency index

{ price, weight etc. satisfaction feelings, preferences

from Eigen vectors from Eigen value

For more visit http://bpmsg.com
Analytic Hierarchy Process

Step 1: Define Objective

Step 2: Structure elements in criteria, sub-criteria, alternatives etc.

Step 3: Make a pair wise comparison of elements in each group

Step 4: Calculate weighting and consistency ratio

Step 5: Evaluate alternatives according weighting

Get ranking

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Analytic Hierarchy Process – Example

Objective: Buy a gadget (smart phone, MP3 player…)

Criteria:
- Color: Pink, blue, green, black, red
- Memory: 8 MB, 16 MB, 32 MB, 64 MB
- Delivery: Immediate, 5 days, 4 weeks

Models:
1. Pink, 32 MB, immediate, 120$
2. Blue, 16 MB, immediate, 120$
3. Black, 32 MB, 1 week, 150$
4. Red, 64 MB, 4 weeks, 150$

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AHP

Structure elements in criteria, sub-criteria, alternatives etc.

Objective

Criteria

Sub-Criteria

Gadget to buy

Color

- pink
- blue
- green
- black
- red

Memory

- 8 MB
- 16 MB
- 32 MB
- 64 MB

Delivery

- immediate
- one week
- 4 weeks

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Compare all elements **pair wise** with respect to the objective

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Compare all elements **pair wise** with respect to the objective

Scale:

- **9**: extreme
- **7**: very strong
- **5**: strong
- **3**: moderate strong
- **1**: equal
- **1/3**: moderate strong
- **1/5**: strong
- **1/7**: very strong
- **1/9**: extreme

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### AHP

Compare all elements **pair wise** with respect to the objective

**Scale:**

<table>
<thead>
<tr>
<th>Intensity of importance</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
<td>Two elements contribute equally to the objective</td>
</tr>
<tr>
<td>3</td>
<td>Moderate importance</td>
<td>Experience and judgment slightly favor one element over another</td>
</tr>
<tr>
<td>5</td>
<td>Strong Importance</td>
<td>Experience and judgment strongly favor one element over another</td>
</tr>
<tr>
<td>7</td>
<td>Very strong importance</td>
<td>One element is favored very strongly over another, it dominance is demonstrated in practice</td>
</tr>
<tr>
<td>9</td>
<td>Extreme importance</td>
<td>The evidence favoring one element over another is of the highest possible order of affirmation</td>
</tr>
</tbody>
</table>

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Compare all elements **pair wise** with respect to the objective

- **Color** → **Memory**: $3^{-1}$
  - Memory is 3 times more important than Color
- **Color** → **Delivery**: $\frac{1}{2}$
  - Delivery is 2 times more important than Color
- **Memory** → **Delivery**: 1
  - Memory is equal important to Delivery

$$\frac{n^2 - n}{2}$$

$n = 3$ results in 3 comparisons

For more visit [http://bpmsq.com](http://bpmsq.com)
Arrange the result in a matrix

For more visit [http://bpmsg.com](http://bpmsg.com)
Arrange the result in a matrix

and compute the normalized principal Eigen vector of the matrix

For more visit http://bpmsq.com
Find the Eigen vector of the matrix

Matrix $N$ for $n$ (=3) criteria

$$N = \begin{bmatrix} 1 & a_{12} & a_{13} \\ a_{12}^{-1} & 1 & a_{23} \\ a_{13}^{-1} & a_{23}^{-1} & 1 \end{bmatrix}$$

Sum of columns

$$\begin{bmatrix} S_{C1} \\ S_{C2} \\ S_{C3} \end{bmatrix}$$

Normalize and calculate first normalized principal Eigen vector $x_1$

$$|N| = \begin{bmatrix} 1 & a_{12}^{-1} & a_{13}^{-1} \\ S_{C1} & S_{C2}^{-1} & S_{C3}^{-1} \\ a_{12}^{-1} & 1 & a_{23}^{-1} \end{bmatrix}$$

$$x_1 = \begin{bmatrix} \sum_{row \overline{1}} n \\ \sum_{row \overline{2}} n \\ \sum_{row \overline{3}} n \end{bmatrix}$$

Square normalized Matrix $|N|$ and calculate next iteration of Eigen vector until difference $x_{k+1} - x_k$ is neglect able

$$x_2 \rightarrow |N|^2$$

For more visit [http://bpmsg.com](http://bpmsg.com)
Find the Eigen vector of the matrix

\[ \lambda = S_{c1} \cdot x_1 + S_{c2} \cdot x_2 + S_{c3} \cdot x_3 \]

Calculate largest Eigen value \( \lambda \)

Calculate Consistency Index

\[ CI = \frac{\lambda - n}{n - 1} \]

Verify Consistency Ratio < 10%

\[ CR = \frac{CI}{RI} \]

Excel Sheet available for download

Random Index \( RI \)

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>RI</td>
<td>0.00</td>
<td>0.00</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.49</td>
</tr>
</tbody>
</table>

For more visit [http://bpmsq.com](http://bpmsq.com)
AHP

Result:

Gadget to buy

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Weight</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Color</td>
<td>17%</td>
<td>3</td>
</tr>
<tr>
<td>Memory</td>
<td>43%</td>
<td>1</td>
</tr>
<tr>
<td>Delivery</td>
<td>40%</td>
<td>2</td>
</tr>
</tbody>
</table>

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Compare all elements *pair wise* with respect to the objective

Gadget to buy

- **Color**: 17%
  - pink
  - blue
  - green
  - black
  - red

- **Memory**: 43%

- **Delivery**: 40%

For more visit [http://bpmsg.com](http://bpmsg.com)
Compare all elements **pair wise** with respect to the objective

- **pink** → blue: 2
  - green: 3
  - black: 1/2
  - red: 1/4

- blue → green: 3
  - black: 1/4
  - red: 1/3

- green → black: 1/7
  - red: 1/7

- black → red: 1/3

n = 5 results in 10 comparisons

For more visit [http://bpmsg.com](http://bpmsg.com)
Arrange the result in a matrix

<table>
<thead>
<tr>
<th></th>
<th>pink</th>
<th>blue</th>
<th>green</th>
<th>black</th>
<th>red</th>
</tr>
</thead>
<tbody>
<tr>
<td>pink</td>
<td>1</td>
<td>½</td>
<td>3</td>
<td>¼</td>
<td>¼</td>
</tr>
<tr>
<td>blue</td>
<td>½</td>
<td>1</td>
<td>3</td>
<td>¼</td>
<td>3⁻¹</td>
</tr>
<tr>
<td>green</td>
<td>3⁻¹</td>
<td>3⁻¹</td>
<td>1</td>
<td>7⁻¹</td>
<td>7⁻¹</td>
</tr>
<tr>
<td>black</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>1</td>
<td>3⁻¹</td>
</tr>
<tr>
<td>red</td>
<td>4</td>
<td>3</td>
<td>7</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

and compute the normalized principal Eigen vector of the matrix

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Result:

Gadget to buy

- Color (17%)
  - pink (13%)
  - blue (12%)
  - green (5%)
  - black (22%)
  - red (49%)

- Memory (43%)
- Delivery (40%)

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Weight sub-criteria according weights of main-criteria

Gadget to buy

- Color 17%
  - pink 13% 2.2%
  - blue 12% 2.0%
  - green 5% 0.9%
  - black 22% 3.7%
  - red 49% 8.3%

- Memory 43%

- Delivery 40%

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AHP

Complete Result:

- **Color** (17%)
  - Pink: 13% (2.2%)
  - Blue: 12% (2.0%)
  - Green: 5% (0.9%)
  - Black: 22% (3.7%)
  - Red: 49% (8.3%)

- **Memory** (43%)
  - 8 MB: 6% (2%)
  - 16 MB: 7% (3%)
  - 32 MB: 43% (19%)
  - 64 MB: 44% (19%)

- **Delivery** (40%)
  - Immediate: 46% (18%)
  - One Week: 43% (17%)
  - 4 Weeks: 12% (5%)

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Evaluate alternatives

**Gadget to buy**

- **Color 17%**
  - **pink** 13% 2.2%
  - **blue** 12% 2.0%
  - **green** 5% 0.9%
  - **black** 22% 3.7%
  - **red** 49% 8.3%

- **Memory 43%**
  - 8 MB 6% 2%
  - 16 MB 7% 3%
  - 32 MB 43% 19%
  - 64 MB 44% 19%

- **Delivery 40%**
  - Immediate 46% 18%
  - One week 43% 17%
  - 4 weeks 12% 5%

**Alternatives**

- **Model 1** Pink, 32 MB, immediate 39%
- **Model 2** Blue, 16 MB, immediate 23%
- **Model 3** Black, 32 MB, 1 week 40%
- **Model 4** Red, 64 MB, 4 weeks 32%

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Evaluate alternatives

Gadget to buy

Color 17%
- Pink: 13%, 2.2%
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- 4 weeks: 12%, 5%

Alternatives
- Model 1: Pink, 32 MB, immediate 39%
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- Model 4: Red, 64 MB, 4 weeks 32%

Benefit
- 2% + 19%

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AHP

Evaluate alternatives

Gadget to buy

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Evaluate alternatives

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Memory 43%
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Alternatives
- Model 1: Pink, 32 MB, immediate 39%
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Benefit
- 2% + 19% + 18% = 39%

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Evaluate alternatives

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Alternatives

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Model 3 has the highest ranking

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AHP – cost vs. benefit

For more visit http://bpmsg.com
Model 3 has similar benefits compared to model 1, but higher costs. Probably you would go for model 1 with immediate delivery and lower price.

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Model 4 has significant higher benefits than model 2. Probably you would go for model 4 accepting longer delivery and higher price.

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Applications:

- Evaluation of product features
- Cost-Benefit Analysis
- Strategy development
- Selection of Key Performance Indicators
- Weighting of objectives in MBOs
- Decision making with multiple inputs from different stakeholders ...

Thank You!

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