Analytic Network Process (ANP)

Klaus D. Goepel Feb. 2011
The Analytic Network Process ANP is a decision making method developed by Prof. Thomas L. Saaty. It is a decision finding method.
Analytic Network Process (ANP)
Developed by Prof. Thomas L. Saaty

1. The Analytic Network Process ANP is a **decision making method**

2. ANP is a generalization of the **Analytic Hierarchy Process AHP**

**Overview**
The analytic network process ANP is a decision finding method and generalization of the analytic hierarchy process AHP.
Analytic Network Process (ANP)
Developed by Prof. Thomas L. Saaty

1. The Analytic Network Process ANP is a **decision making method**

2. ANP is a generalization of the **Analytic Hierarchy Process AHP**

Overview
The analytic network process ANP is a decision finding method and generalization of the analytic hierarchy process AHP.

AHP
- Goal
  - Criteria
    - Sub-criteria
      - Alternatives

ANP
- Control Criterion
  - Cluster 1
    - Cluster 2
      - Alternatives
The Analytic Network Process ANP is a decision making method and a generalization of the Analytic Hierarchy Process AHP.

ANP can model complex decision problems, where a hierarchical model – as used in AHP – is not sufficient.
The Analytic Network Process ANP is a decision making method.

ANP is a generalization of the Analytic Hierarchy Process AHP.

ANP can model complex decision problems.

It allows for feedback connections and loops.

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ANP can model complex decision problems, where a hierarchical model – as used in AHP – is not sufficient.

ANP allows for feedback connections and loops.
Analytic Network Process (ANP)

Overview

Example

Decision for the selection of a candidate in recruitment of a sales engineer
Analytic Network Process (ANP)

Overview

In AHP you do a pair-wise comparison of criteria and sub-criteria, resulting in local priorities or weighting factors.
Analytic Network Process (ANP)

Overview

In ANP you do a pair-wise comparison of criteria and sub-criteria, resulting in local priorities or weighting factors.

By applying the global priorities to alternatives, you finally get a ranking of alternatives with respect to these criteria and sub-criteria.

It's a top-down structure from the overall objective to criteria, from criteria to sub-criteria down to alternatives.
Analytic Network Process (ANP)

In ANP criteria, sub-criteria and alternatives are treated equally as nodes in a network. Each of these nodes might be compared to any other node, as long as there is a relation between them.
Analytic Network Process (ANP)

Control Criteria

Criteria

Alternatives

Overview

In ANP criteria, sub-criteria and alternatives are treated equally as nodes in a network.

Each of these nodes might be compared to any other node, as long as there is a relation between them.

For example, the ranking of alternatives might not only depend on the weighting of criteria, but also given alternatives can influence the ranking of criteria.

Given Alternatives can influence the weighting of criteria
In contrast to AHP, where higher level elements connect to lower levels – i.e. criteria to sub-criteria – in ANP nodes might be grouped in clusters.

Beside local priorities in the comparison of one node to a set of other nodes, you might also introduce cluster priorities with respect to the goals.
### Analytic Network Process (ANP)

The network of ANP is represented as a matrix. The matrix is composed by listing all nodes horizontally and vertically.

#### The Super Matrix

The network of ANP is represented as a matrix.

- The matrix is composed by listing all nodes horizontally and vertically,
Analytic Network Process (ANP)

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Each non-zero element of the matrix represents the connection & weight from one node (columns-header) to another node (row-header) of the network.
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### The Network of ANP

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<td></td>
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<tr>
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- Technical
- Sales
- Experience
- Hire
- Altern.

Candidate 1

Candidate 2
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Diagram:

- Hire
  - Technical
  - Sales
  - Experience
- Altern.
  - Candidate 1
  - Candidate 2

Diagram:

- Hire
  - Technical
  - Sales
  - Experience
- Altern.
  - Candidate 1
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The matrix is called Super-Matrix
The Super Matrix

The comparison of nodes – connected to others – follows the same principal and method as in AHP.

Analytic Network Process (ANP)
Analytic Network Process (ANP)

The Super Matrix

The comparison of nodes – connected to others – follows the same principal and method as in AHP. Local priorities result from the Eigenvector of the comparison matrix. The so found priorities are then arranged as column vectors in the super-matrix.

Comparison Matrix wrt Hiring

Technical | Sales | Experience
--- | --- | ---
Technical | 1/2 | 1/3
Sales | 2 | 1/3
Experience | 3 | 3

Hierarchical Model

The Super Matrix

Priority Vector resulting from pair-wise comparisons

Comparison of Criteria wrt Hire:

Sales Skills are equally to moderately more important than Technical Skills (2x)

Experience is moderately more important than Technical Skills (3x)

Experience is moderately more important than Sales Skills (3x)
Analytic Network Process (ANP)

The Super Matrix

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The so found priorities are then arranged as column vectors in the super-matrix.

Comparison of Candidates wrt Technical Skills

Candidate 1 has equally technical Skills as Candidate 2 (1)
Analytic Network Process (ANP)

The Super Matrix

The comparison of nodes – connected to others – follows the same principal and method as in AHP. Local priorities result from the Eigenvector of the comparison matrix. The so found priorities are then arranged as column vectors in the super-matrix.

Comparison of Candidates wrt Sales Skills

Candidate 2 has moderately to strongly better Sales Skills than Candidate 1 (4x)
### Analytic Network Process (ANP)

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### The Super Matrix

### Comparison of Candidates wrt Experience

Candidate 1 has equally to Moderately better Experience than Candidate 2 (2x)

### Hierarchy Model

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<tbody>
<tr>
<td>Candidate 1</td>
<td>50</td>
<td>16</td>
<td>50</td>
<td>50</td>
<td>59</td>
</tr>
<tr>
<td>Candidate 2</td>
<td>80</td>
<td>25</td>
<td>20</td>
<td>80</td>
<td>33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Altern.</th>
<th>Candidate 1</th>
<th>50</th>
<th>2</th>
<th>67</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Candidate 2</td>
<td>50</td>
<td>1/2</td>
<td>33</td>
</tr>
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</table>

**Comp. Matrix wrt Experience**

- Candidate 1: 0.67
- Candidate 2: 0.33

**Candidate 1 has equally to Moderately better Experience than Candidate 2 (2x)**
Analytic Network Process (ANP)

The Super Matrix

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The Analytic Network Process (ANP)

Impact of Alternatives on the priorities of criteria

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The Super Matrix

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Comparison of Criteria wrt Candidate 1

Technical Skills are strongly to very strongly more prevalent than Sales Skills (6x)

Technical Skills are strongly to very strongly more prevalent than Experience (6x)

Sales Skills are equally to Experience (1)
The Analytic Network Process (ANP)

The Super Matrix

The comparison of nodes – connected to others – follows the same principal and method as in AHP. Local priorities result from the Eigenvector of the comparison matrix. The so found priorities are then arranged as column vectors in the super-matrix.

Comparison of Criteria wrt Candidate 2

Sales Skills are strongly to very strongly more prevalent than Technical Skills (6)

Sales Skills are strongly to very strongly more prevalent than Experience (6)

Technical Skills are equally to Experience (1)
Analytic Network Process (ANP)

The Super Matrix

After all comparisons are done, we get the "Unweighted Super Matrix"

This matrix is then normalized i.e. the sum of all columns is scaled to 1
Analytic Network Process (ANP)

The Super Matrix

After all comparisons are done, we get the “Unweighted Super Matrix”

This matrix is then normalized i.e. the sum of all columns is scaled to 1

The whole model is synthesized by calculating the “Limit Matrix”. The Limit Matrix is the weighted Super matrix, taken to the power of \( k+1 \), where \( k \) is an arbitrary number.
Both candidates have the required experience.

Why changes the ranking of Candidate 2 from two in the Hierarchy model to one in the Network model?
Analytic Network Process (ANP)

Overview

Both candidates have the required experience
- candidate 1 slightly more than candidate 2.

Why changes the ranking of Candidate 2 from two in the Hierarchy model to one in the Network model?
### Analytic Network Process (ANP)

#### Overview
Both candidates have the required experience - candidate 1 slightly more than candidate 2.
Experience is given a relative high weight

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### Weighted Super Matrix

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Candidate 1: 50, 20, 67
Candidate 2: 50, 80, 33

### Why changes the ranking of Candidate 2 from two in the Hierarchy model to one in the Network model?
Analytic Network Process (ANP)

Overview
Both candidates have the required experience - candidate 1 slightly more than candidate 2. Experience is given a relative high weight. Resulting in the slightly higher ranking for candidate 1 in the hierarchical model.

Weighted Super Matrix

Why changes the ranking of Candidate 2 from two in the Hierarchy model to one in the Network model?
Analytic Network Process (ANP)

Overview

In the network model we also look at each candidate’s skills independent from the other candidate.

Why changes the ranking of Candidate 2 from two in the Hierarchy model to one in the Network model?
In the network model we also look at each candidate’s skills independent from the other candidate. Now we see the outstanding sales skills of candidates 2.

Why changes the ranking of Candidate 2 from two in the Hierarchy model to one in the Network model?
Analytic Network Process (ANP)

Overview

In the network model we also look at each candidate’s skills independent from the other candidate.

Now we see the outstanding sales skills of candidate 2.

Finally in the network model, sales skills get more weight in the decision than the excellent technical skills of candidate 1.

Why changes the ranking of Candidate 2 from \textit{two} in the Hierarchy model to \textit{one} in the Network model?
## Analytic Network Process (ANP)

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Why changes the ranking of Candidate 2 from two in the Hierarchy model to one in the Network model?
A decision network can be arranged under a control hierarchy of benefits and costs. In our example here, we already evaluated the benefits of hiring – with respect to the hard – and soft-skills of the two candidates. We can now evaluate the requested salary of both candidates under control criterion costs. Depending on our overall objective, either benefits or costs could be assigned a higher weighting.
We have now a two layer model with a control hierarchy – benefits and costs – and a sub-network under benefits and a hierarchy under costs.

Ranking of alternatives in a two layer model can be evaluated using a ratio formula Benefit/Cost or an additive formula (B-C).

**Evaluation Formulas**

- Benefits/Costs (B/C)
- Benefits - Costs (B-C)
The control hierarchy could be extended with additional control parameter, e.g. opportunities and risks, to build a two layer BOCR model.

Evaluation Formulas

\[
\frac{(B \times O)}{C \times R} \quad \text{and} \quad (B + O) - (C - R)
\]
Thank You!