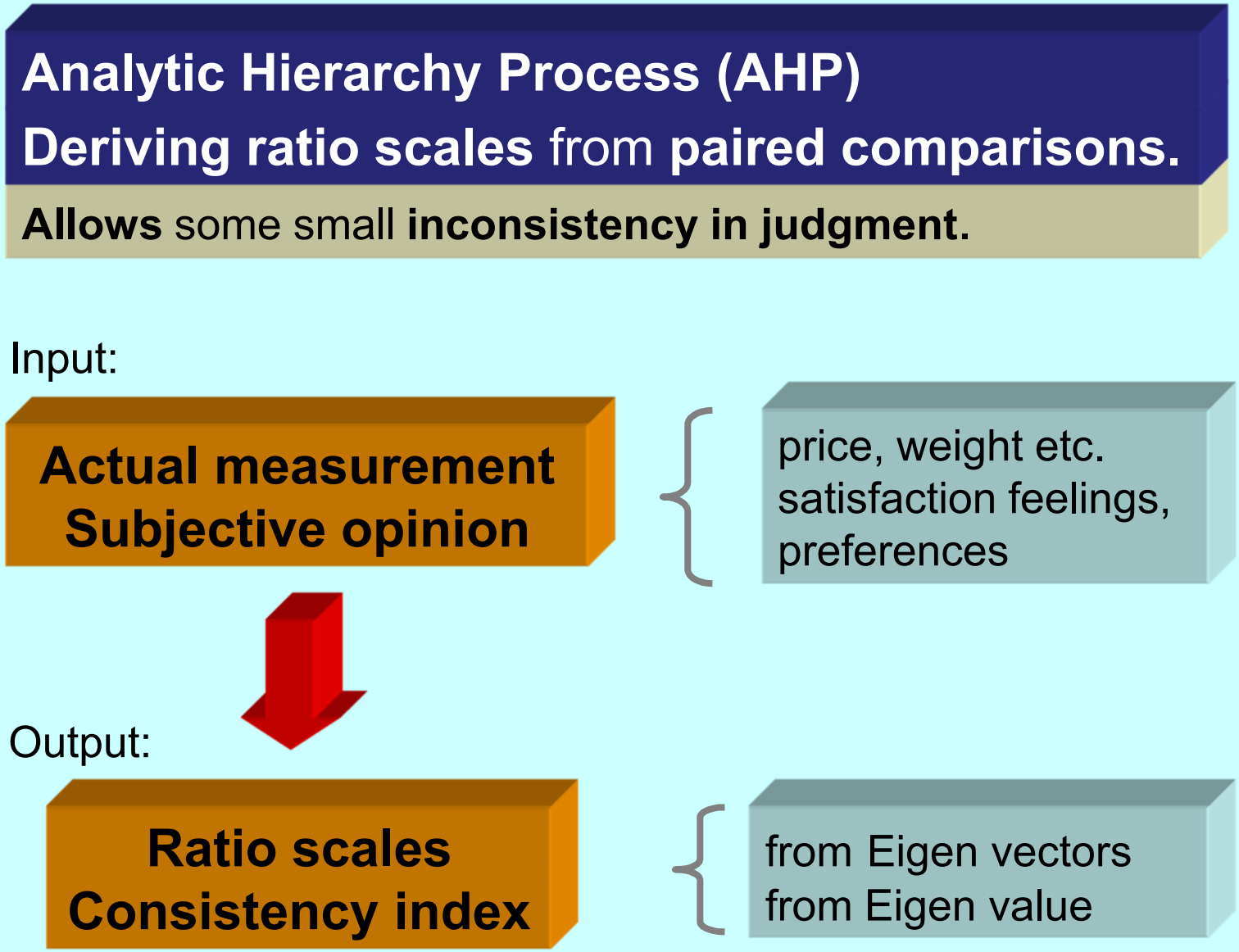
Business Performance Management

Analytic Hierarchy Process

- Multi Criteria decision making method
- Originally developed by Prof. Thomas L. Saaty

Klaus Goepel, Mar. 2010



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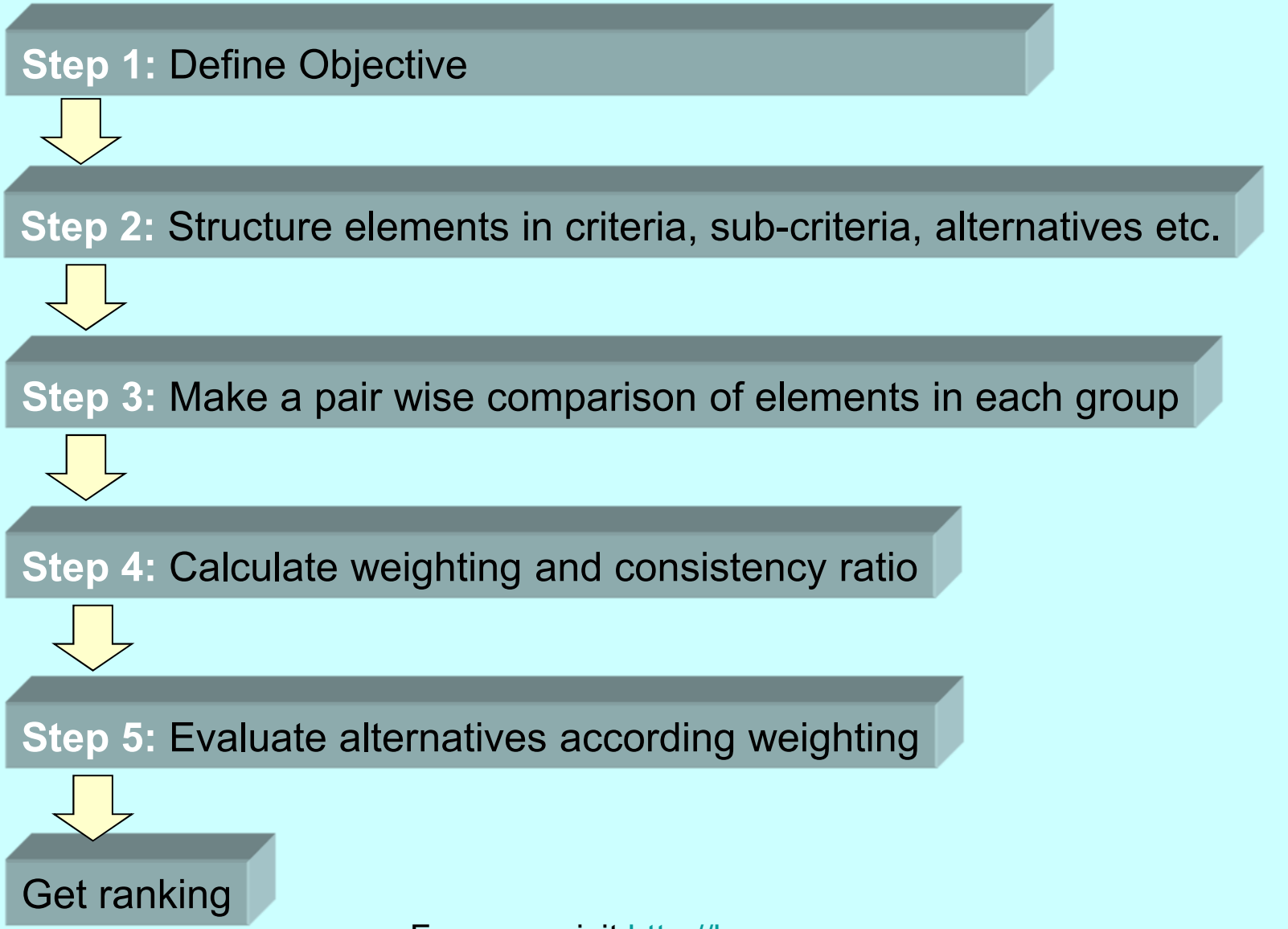
Analytic Hierarchy Process



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Analytic Hierarchy Process – Example

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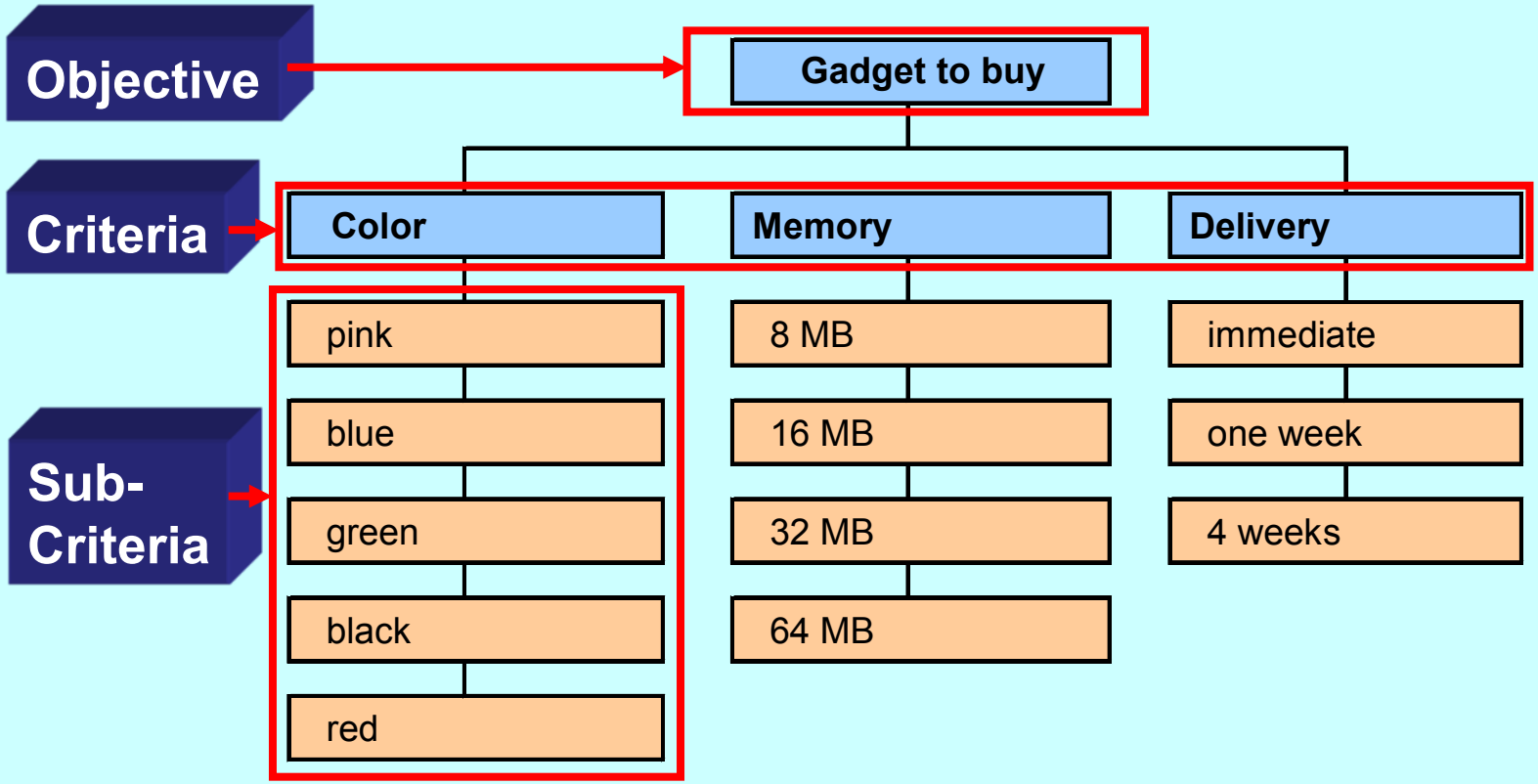
Structure elements in criteria, sub-criteria, alternatives etc.



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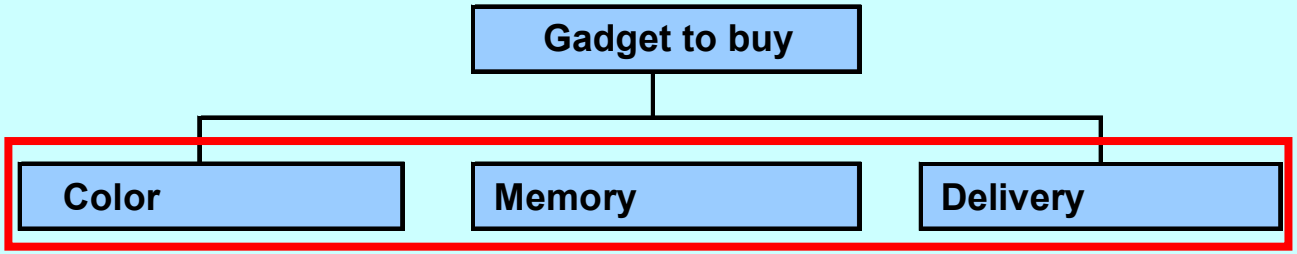
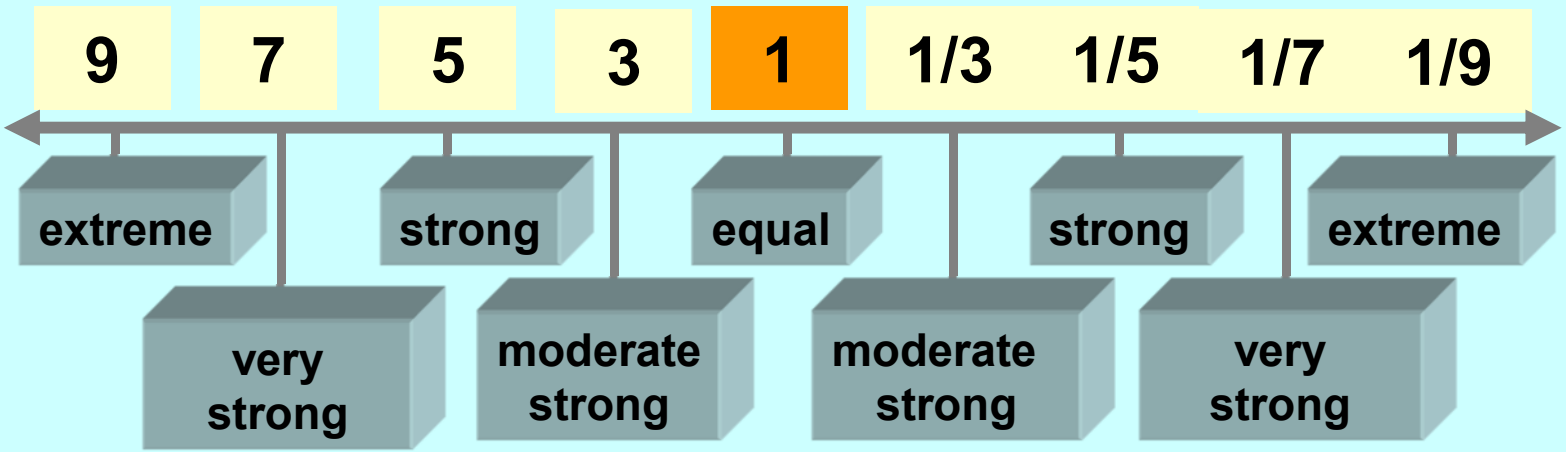
Compare all elements **pair wise** with respect to the objective



AHP

Compare all elements **pair wise** with respect to the objective

Scale:



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Compare all elements **pair wise** with respect to the objective

Scale:

Intensity of importance	Definition	Explanation
1	Equal importance	Two elements contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favor one element over another
5	Strong Importance	Experience and judgment strongly favor one element over another
7	Very strong importance	One element is favored very strongly over another, it dominance is demonstrated in practice
9	Extreme importance	The evidence favoring one element over another is of the highest possible order of affirmation

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Compare all elements **pair wise** with respect to the objective

Color →	*Memory*	3^{-1}	Memory is 3 times more important than Color
Color →	*Delivery*	½	Delivery is 2 times more important than Color
Memory →	*Delivery*	1	Memory is equal important to Delivery

$$\frac{n^2 - n}{2}$$

n = 3 results in 3 comparisons

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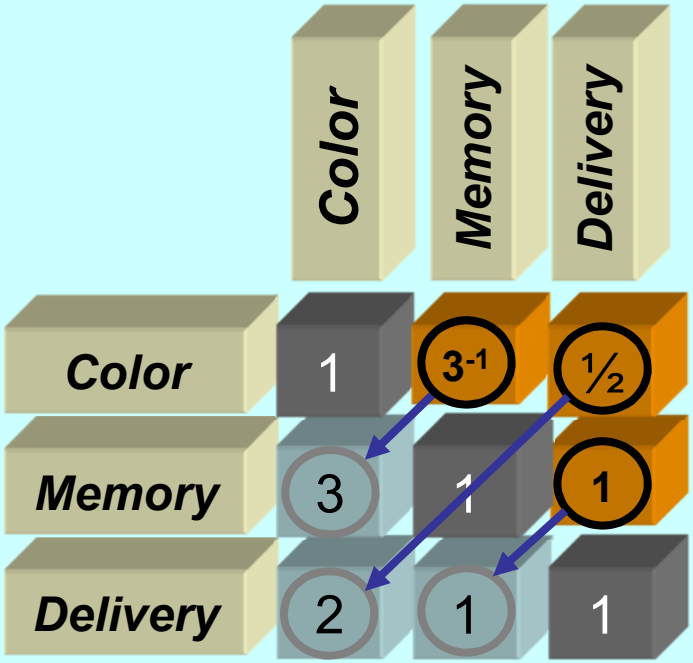
Arrange the result in a matrix



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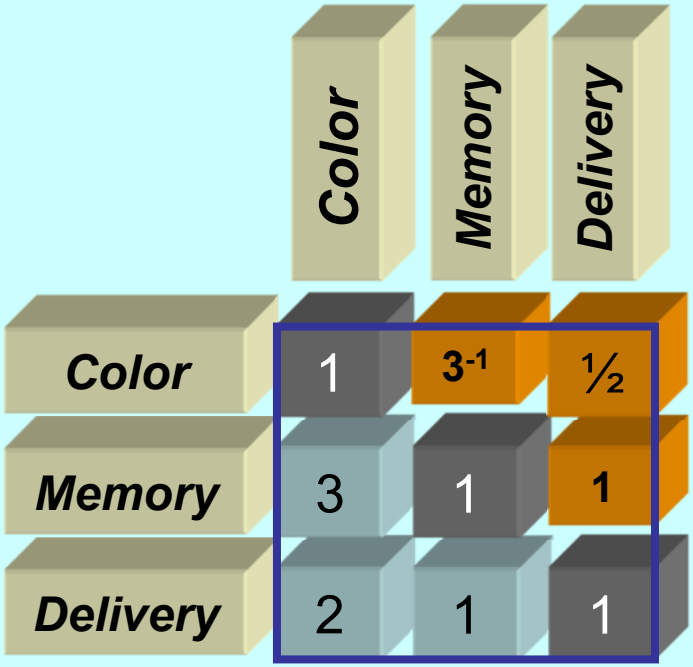
Arrange the result in a matrix



and compute the normalized principal Eigen vector of the matrix

Find the Eigen vevtor of the matrix

Matrix **N** for *n* (=3) criteria

$$\mathbf{N} = \begin{bmatrix} 1 & a_{12} & a_{13} \\ a_{12}^{-1} & 1 & a_{23} \\ a_{13}^{-1} & a_{23}^{-1} & 1 \end{bmatrix}$$

Sum of columns

$$\begin{matrix} S_{C1} & S_{C2} & S_{C3} \end{matrix}$$

Excel Sheet available for download

Normalize and calculate first normalized principal **Eigen vector $\mathbf{x}_1$**

$$|\mathbf{N}| = \begin{bmatrix} \frac{1}{S_{C1}} & \frac{a_{12}}{S_{C2}} & \frac{a_{13}}{S_{C3}} \\ \frac{a_{12}^{-1}}{S_{C1}} & \frac{1}{S_{C2}} & \frac{a_{23}}{S_{C3}} \\ \frac{a_{13}^{-1}}{S_{C1}} & \frac{a_{23}^{-1}}{S_{C2}} & \frac{1}{S_{C3}} \end{bmatrix} \quad \mathbf{x}_1 = \begin{bmatrix} \frac{\sum \text{row}_1}{n} \\ \frac{\sum \text{row}_2}{n} \\ \frac{\sum \text{row}_3}{n} \end{bmatrix}$$

Square normalized Matrix **|N|** and calculate next iteration of Eigen vector until difference $\mathbf{x}_{k+1}$ - $\mathbf{x}_k$ is neglect able

$$\mathbf{x}_2 \rightarrow |\mathbf{N}|^2$$

Find the Eigen vevtor of the matrix

Calculate largest Eigen value λ

$$\lambda = S_{C1} \cdot x_1 + S_{C2} \cdot x_2 + S_{C3} \cdot x_3$$

Calculate Consistency Index

$$CI = \frac{\lambda - n}{n - 1}$$

Excel Sheet available for downlaod

Verify Consistency Ratio < 10%

$$CR = \frac{CI}{RI}$$

Random Index *RI*

n	1	2	3	4	5	6	7	8	9	10
RI	0,00	0,00	0,58	0,90	1,12	1,24	1,32	1,41	1,45	1,49

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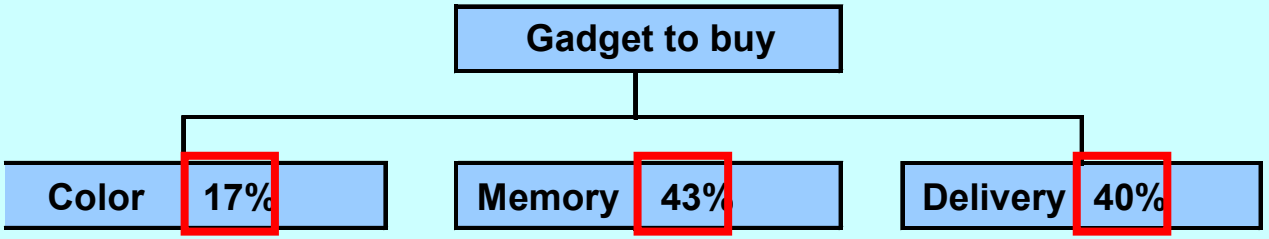
Result:



Criteria:	Weight:	Rank:
Color	17%	3
Memory	43%	1
Delivery	40%	2

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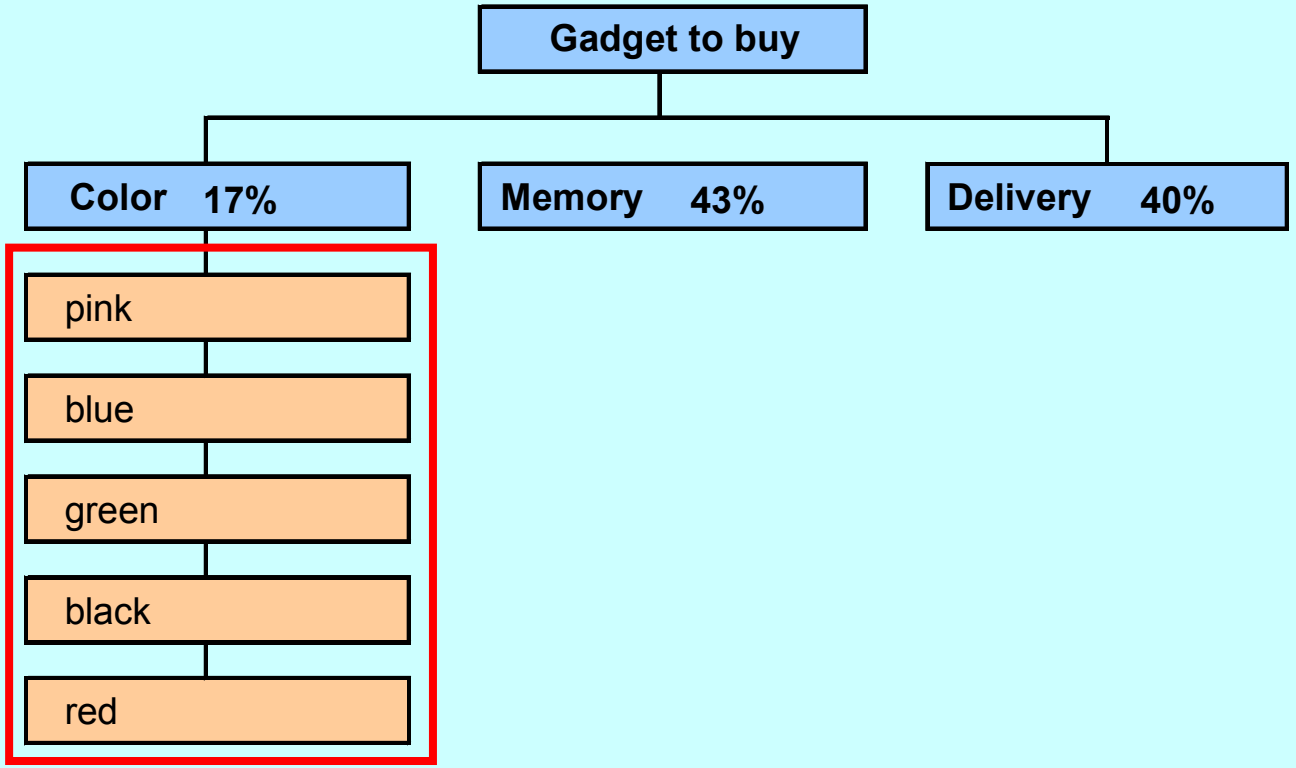
Compare all elements **pair wise** with respect to the objective



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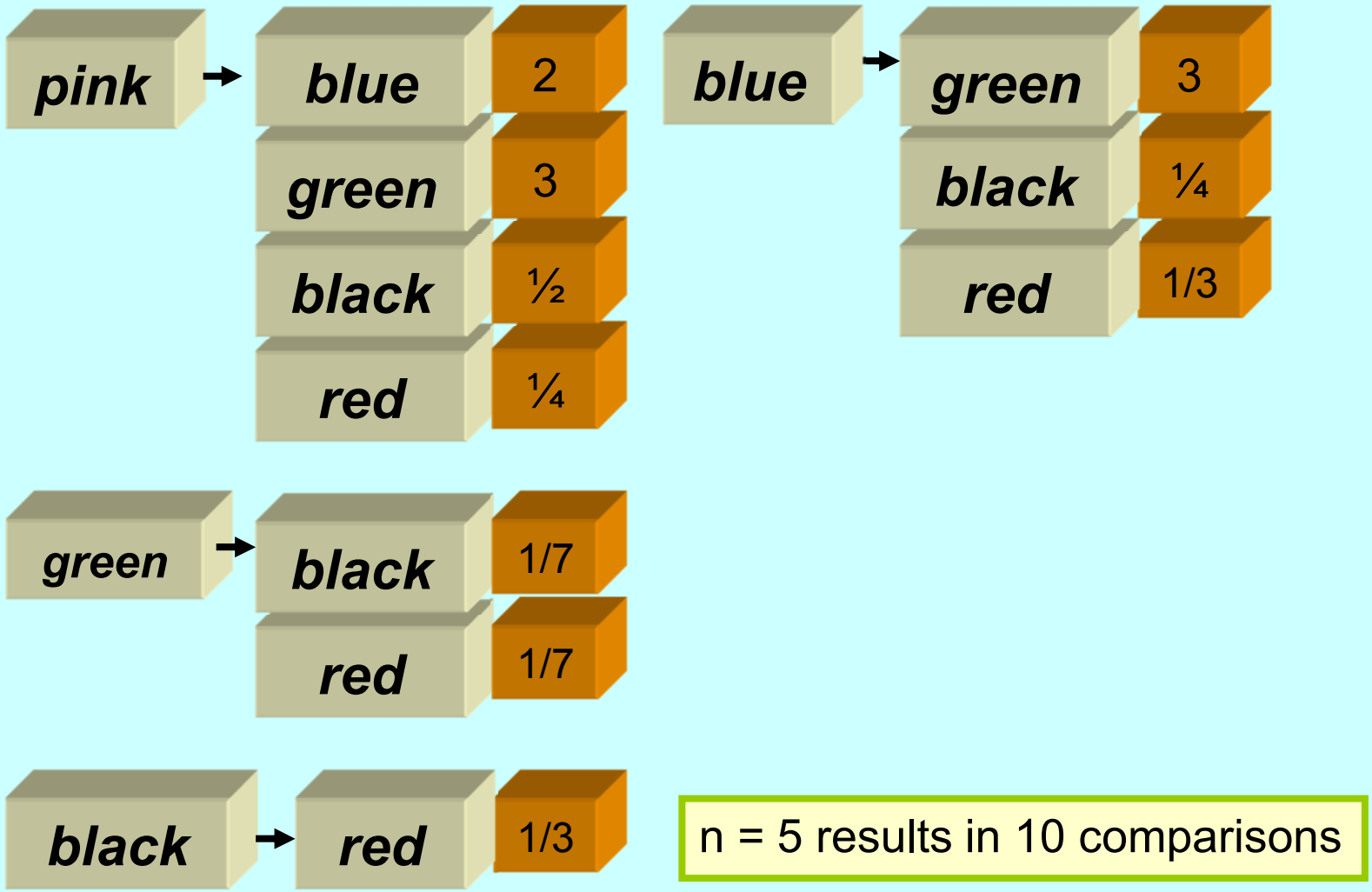
Compare all elements **pair wise** with respect to the objective



n = 5 results in 10 comparisons

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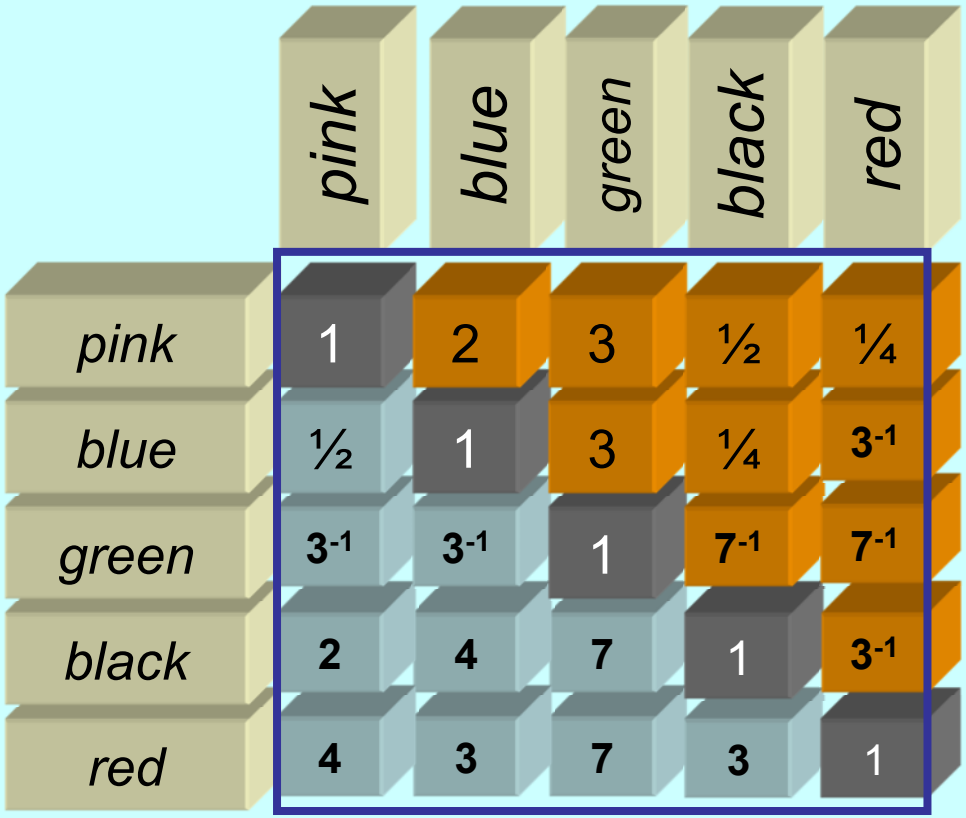
Arrange the result in a matrix



and compute the normalized principal Eigen vector of the matrix

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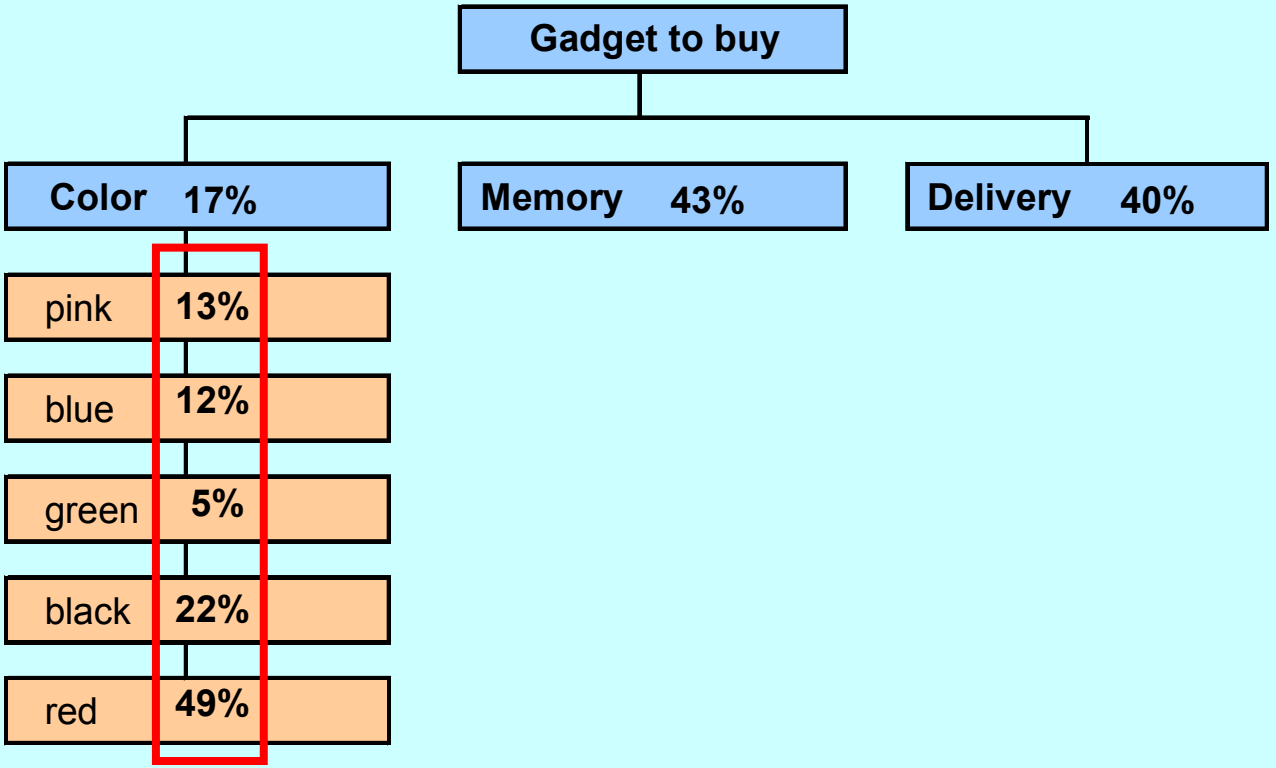
Result:



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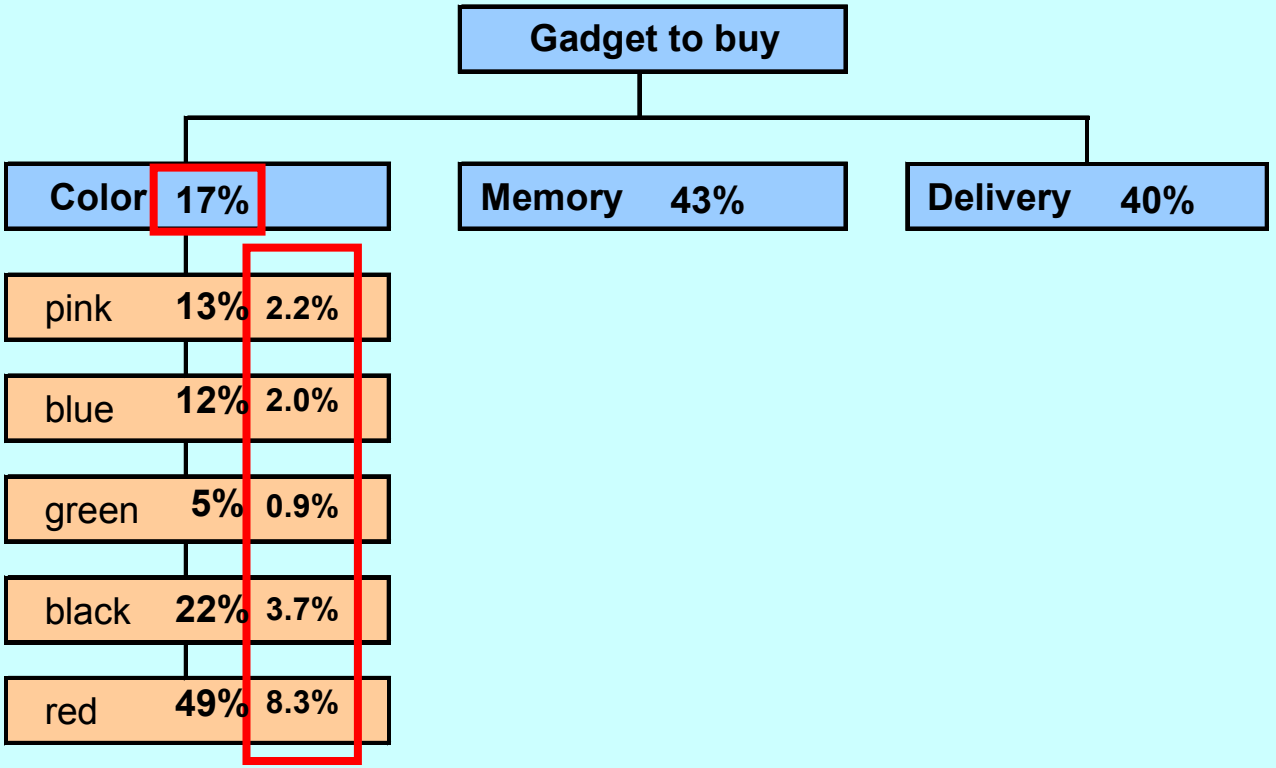
Weight sub-criteria according weights of main-criteria



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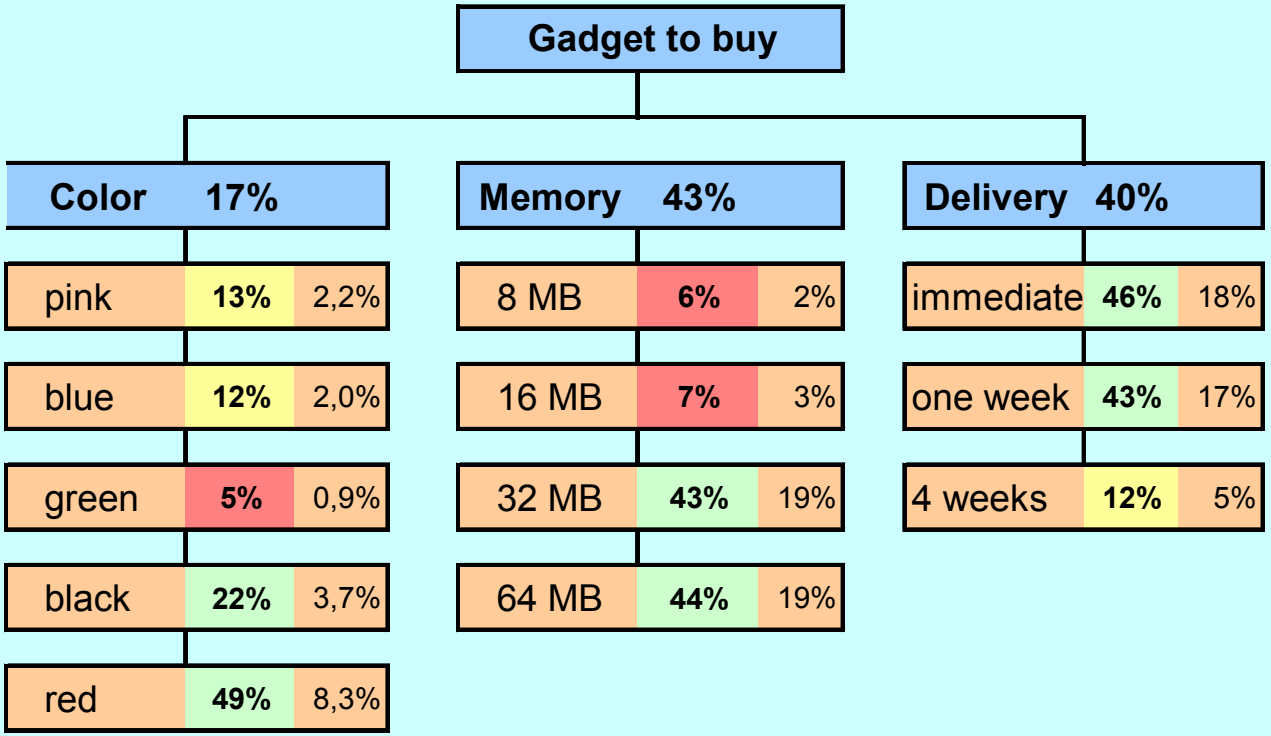
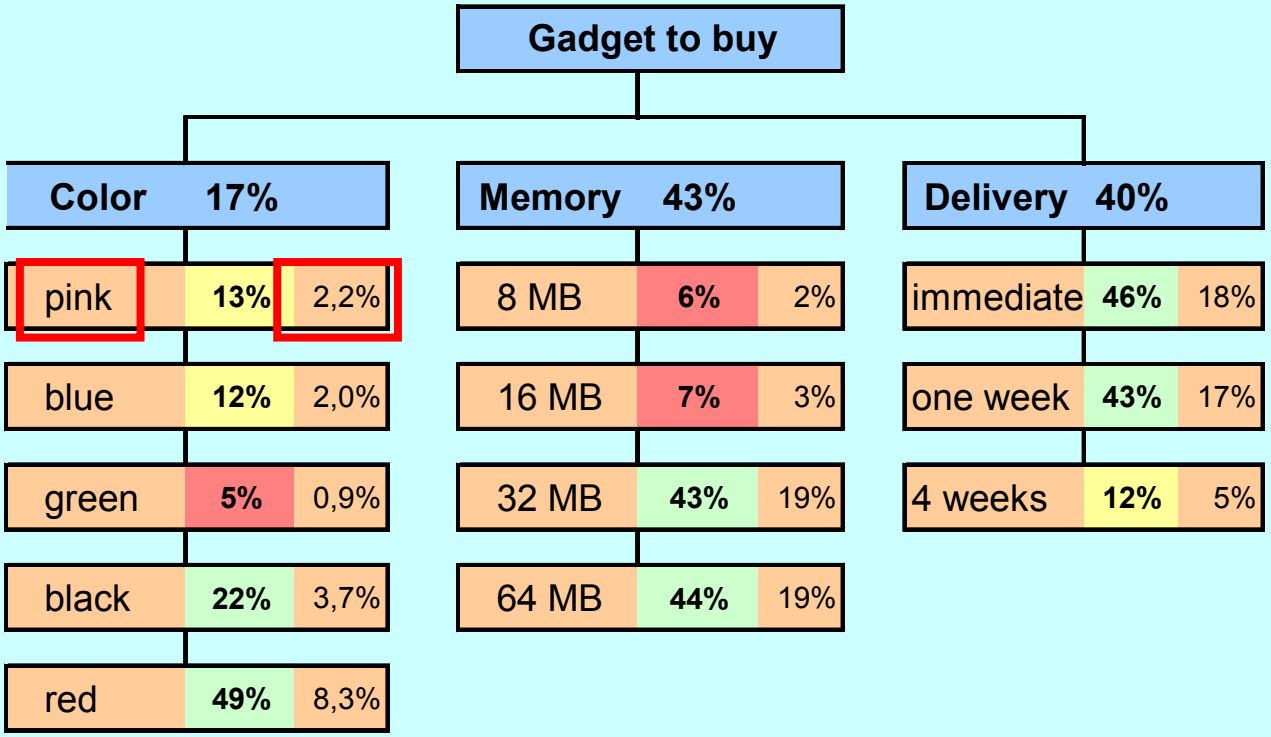
Complete Result:

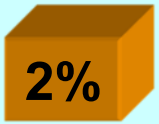


AHP

Evaluate alternatives



Alternatives		Benefit
Model 1	Pink, 32 MB, immediate	**39%**
Model 2	Blue, 16 MB, immediate	**23%**
Model 3	Black, 32 MB, 1 week	**40%**
Model 4	Red, 64 MB, 4 weeks	**32%**



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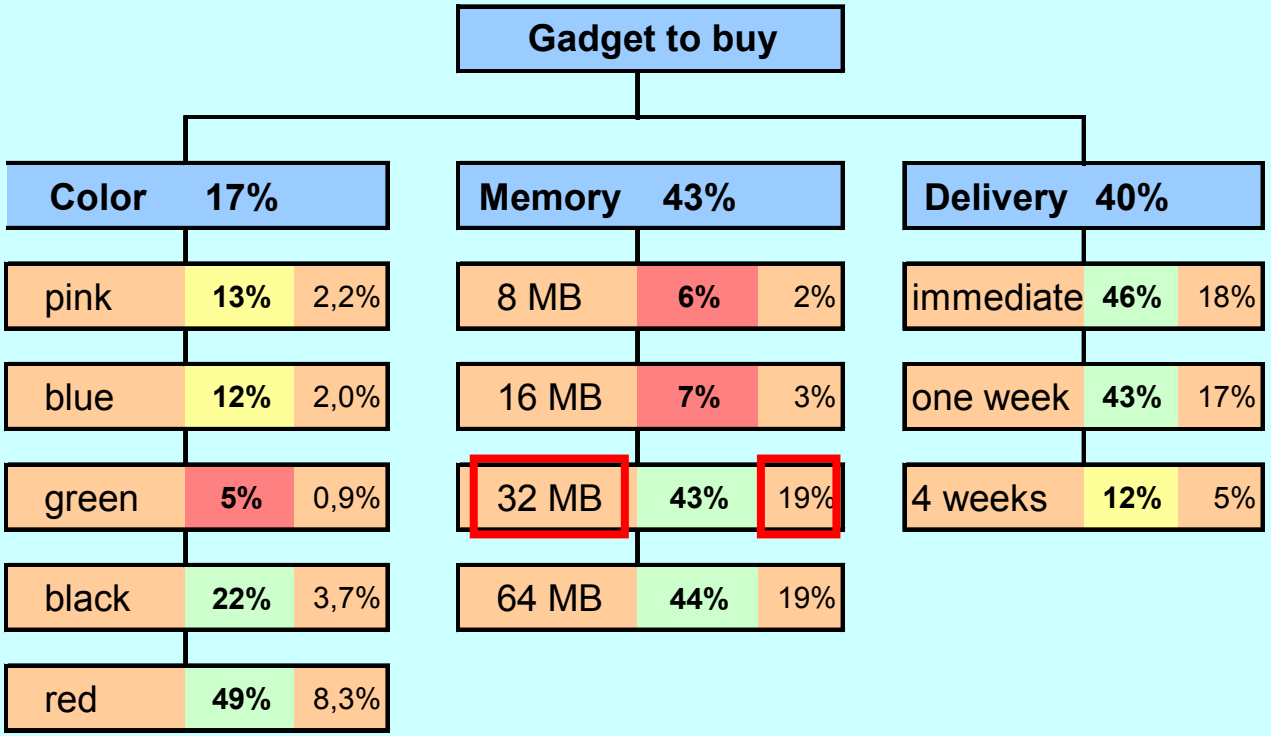
AHP

Evaluate alternatives



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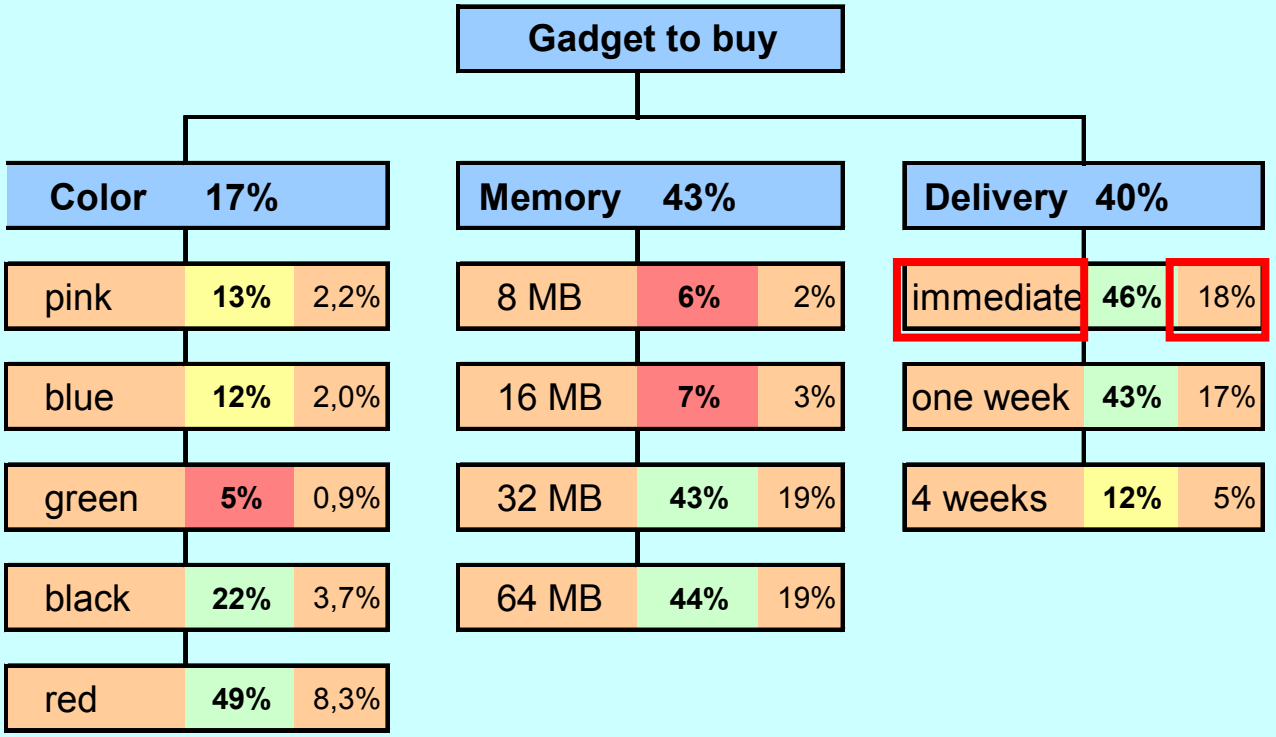
Evaluate alternatives



Alternatives			Benefit
Model 1	Pink, 32 MB	immediate	**39%**
Model 2	Blue, 16 MB	immediate	**23%**
Model 3	Black, 32 MB, 1 week		**40%**
Model 4	Red, 64 MB, 4 weeks		**32%**

2% + 19% + 18%

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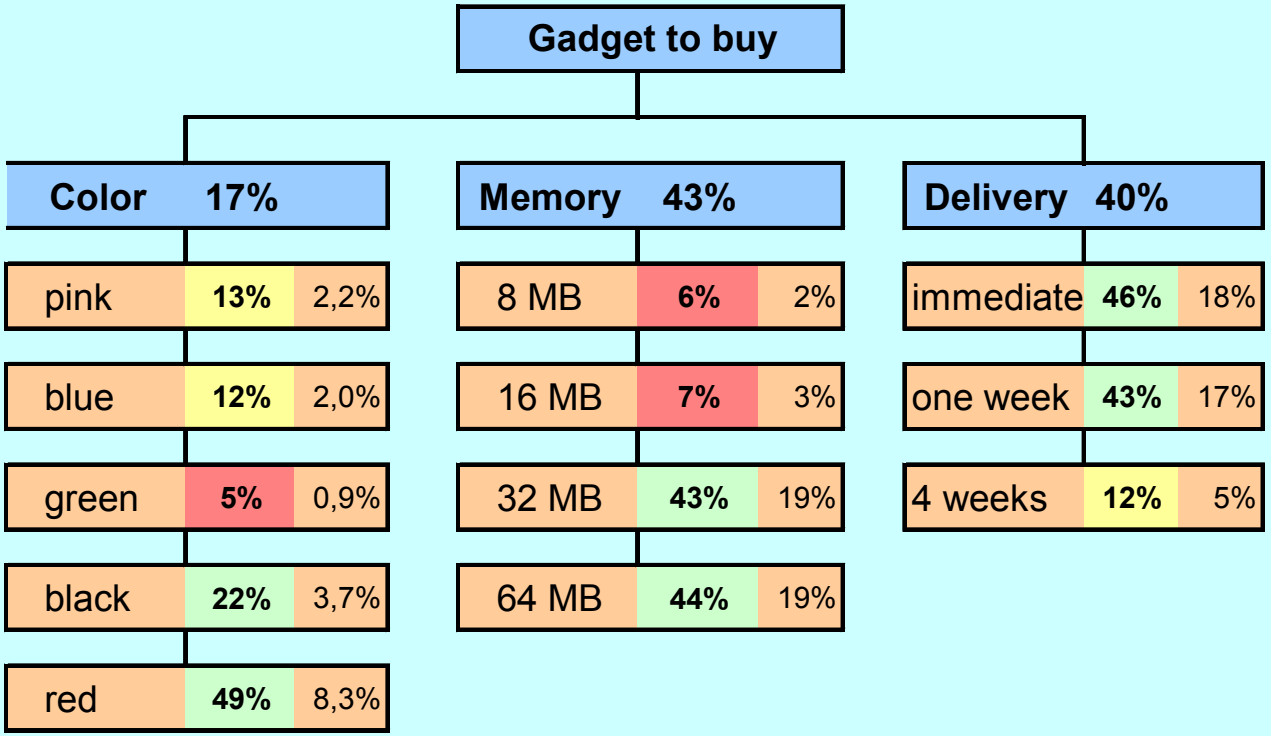
AHP

Evaluate alternatives



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2% + 19% + 18% = 39%

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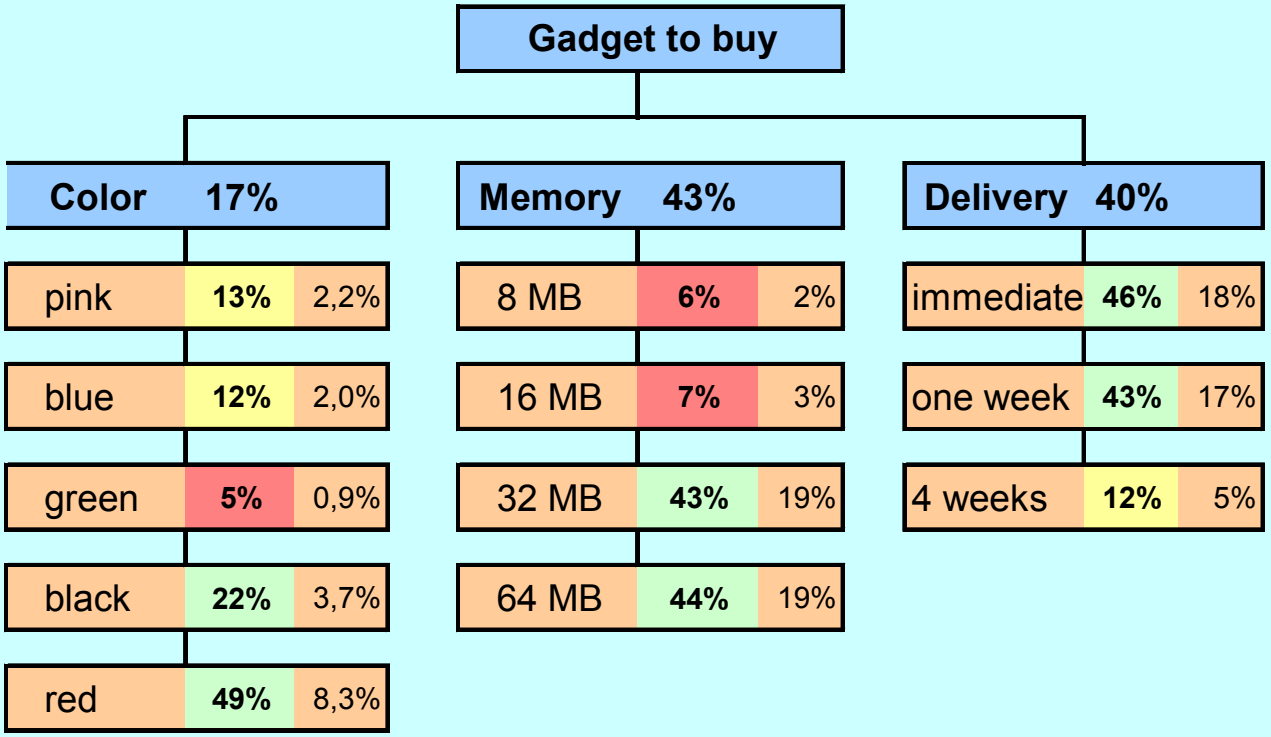
AHP

Evaluate alternatives



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Model 3	Black, 32 MB, 1 week	**40%**
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Model 3 has the highest ranking

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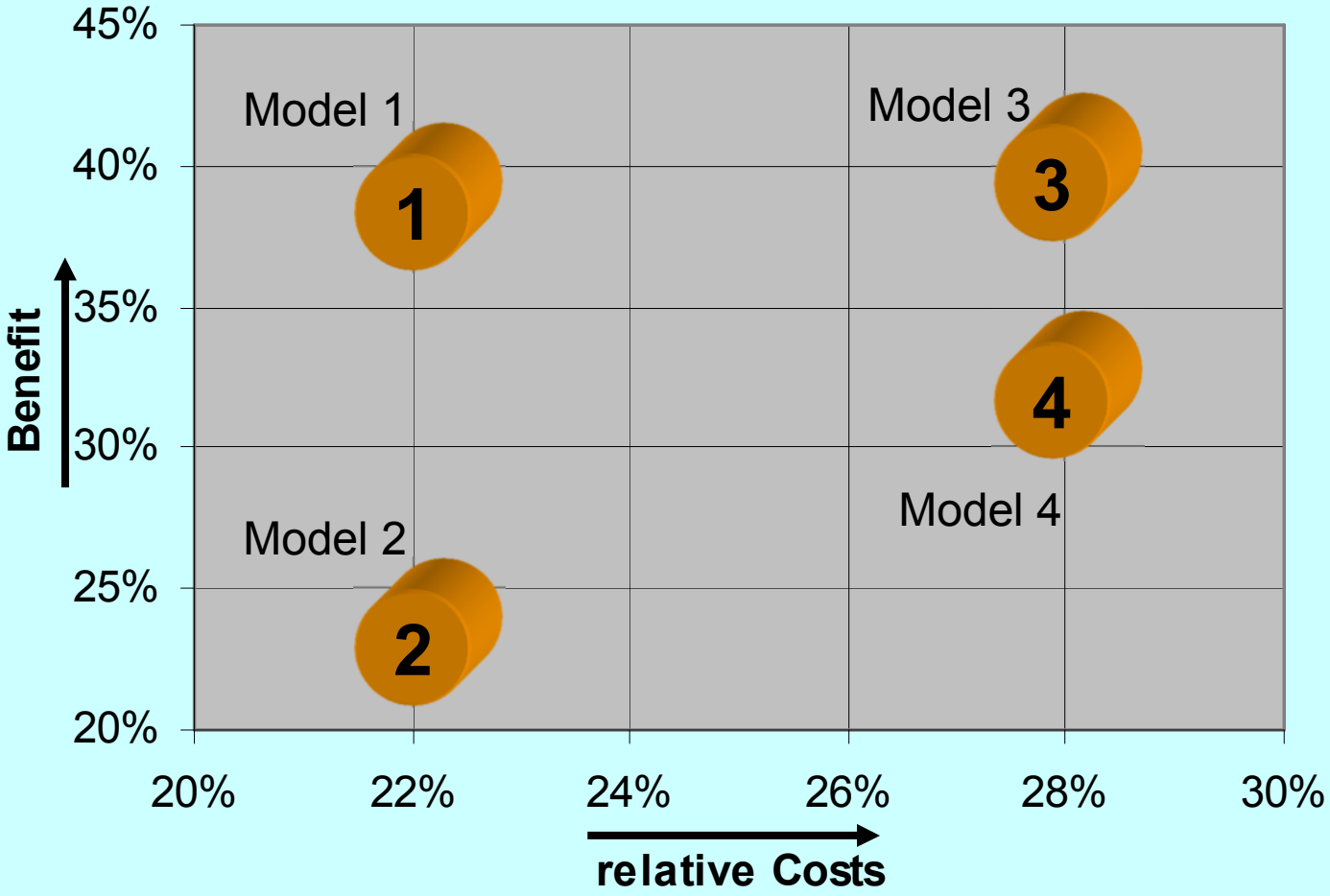
AHP – cost vs. benefit



Alternatives		Benefit	Cost $	Cost (norm)
Model 1	Pink, 32 MB, immediate	**39%**	120	22%
Model 2	Blue, 16 MB, immediate	**23%**	120	22%
Model 3	Black, 32 MB, 1 week	**40%**	150	28%
Model 4	Red, 64 MB, 4 weeks	**32%**	150	28%
			540	

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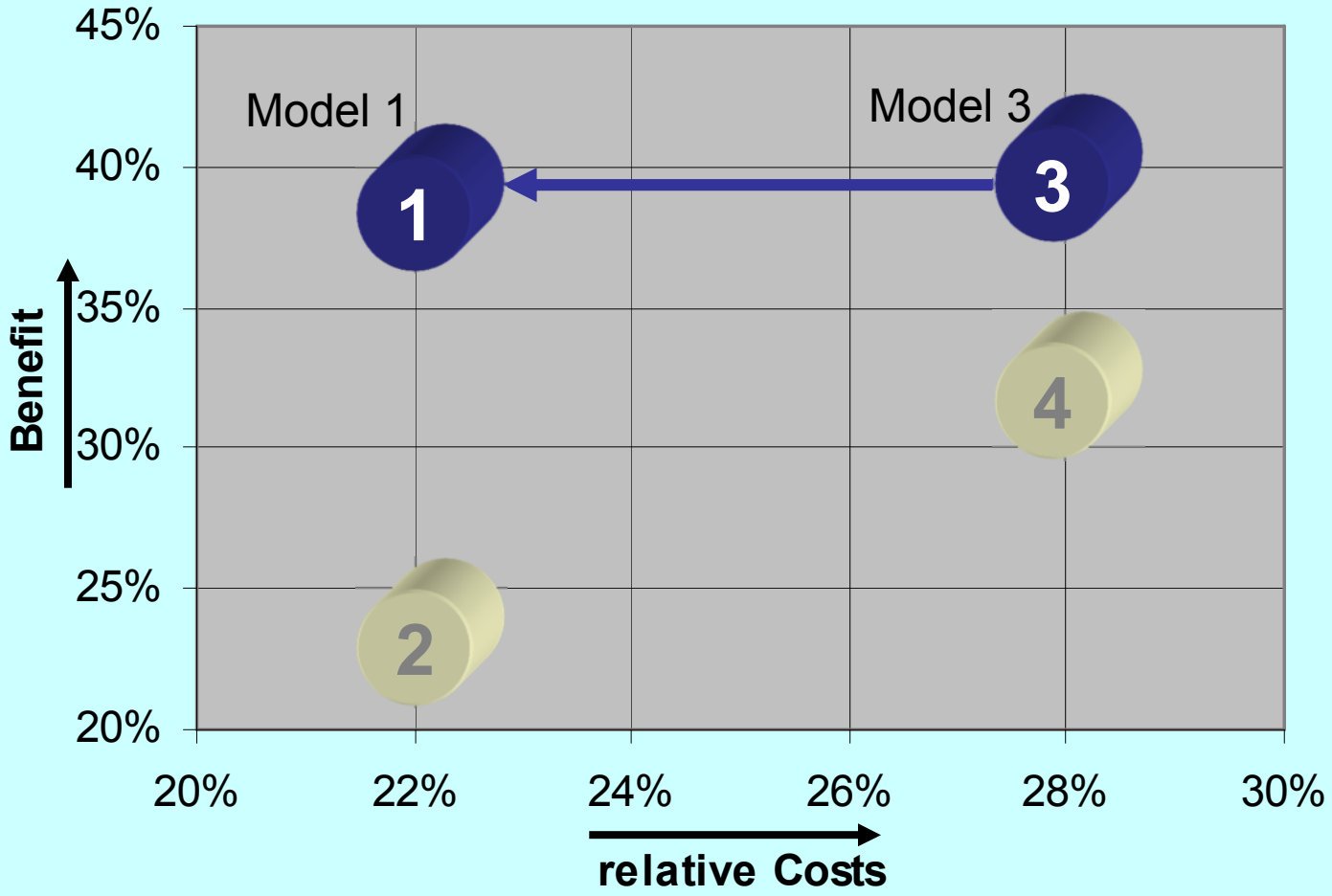
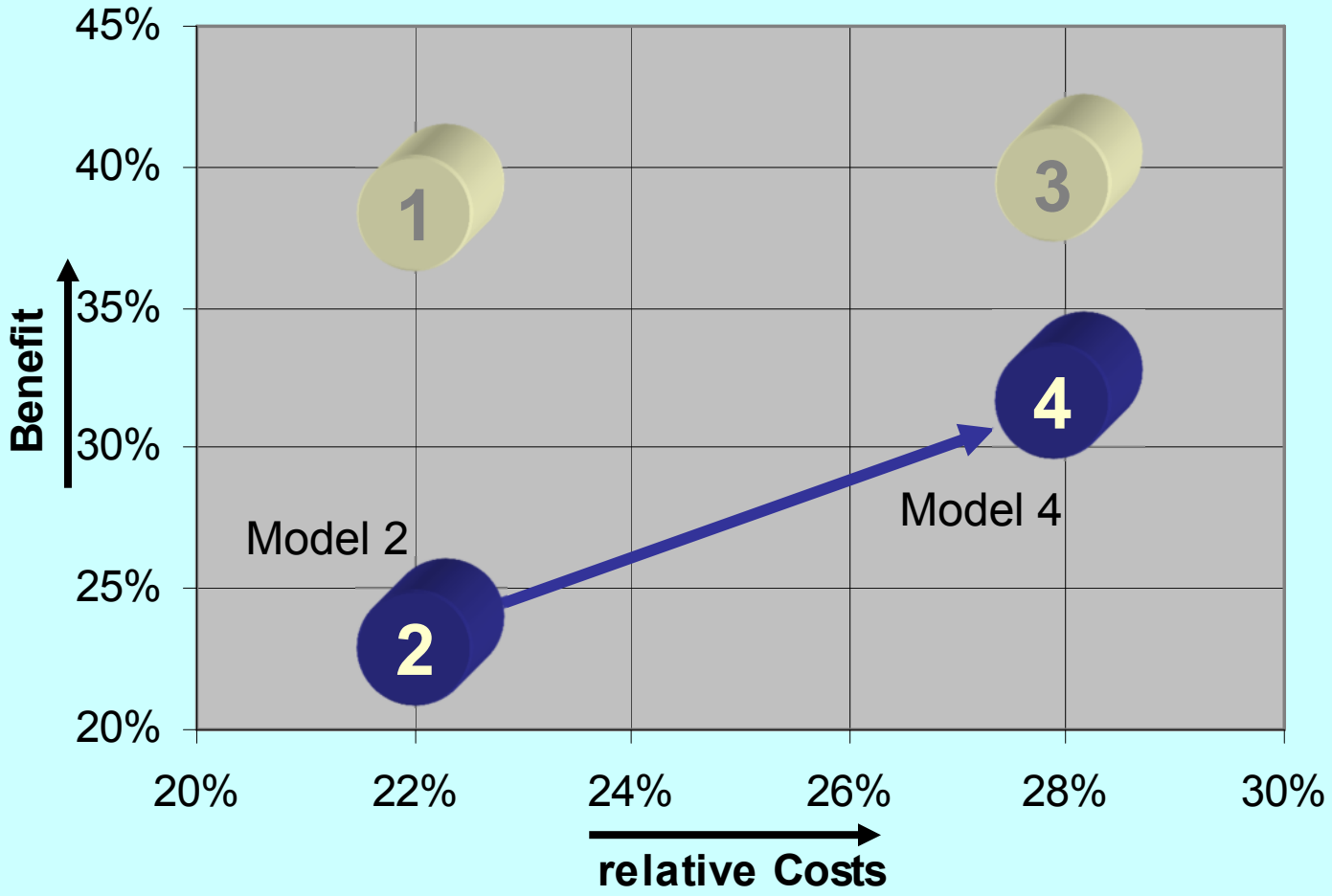
AHP – cost vs. benefit



Model 3 has similar benefits compared to **model 1**, but higher costs. Probably you would go for model 1 with immediate delivery and lower price

AHP – cost vs. benefit



Model 4 has significant higher benefits than **model 2**. Probably you would go for model 4 accepting longer delivery and higher price

AHP

Applications:

- **Evaluation of product features**
- **Cost-Benefit Analysis**
- **Strategy development**
- **Selection of Key Performance Indicators**
- **Weighting of objectives in MBOs**
- **Decision making with multiple inputs from different stakeholders …**

Thank You!

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